

Three Essays in Macroeconomics and Financial Economics

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Three Essays in Macroeconomics and Financial Economics

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Abstract

This dissertation presents empirical analysis of linear and nonlinear models in macroeconomics and financial economics. It conveys the message about the substantial benefit in the analysis stems from a little departure from the standard models. By relaxing some assumptions, especially the linearity, it demonstrates some significant improvements of analysis performed in terms of accuracy and theoretical consistency.

Empirical Analysis of A Core Inflation Measure in An Estimated DSGE Model

The first chapter presents the analysis of inflation by allowing an ad-hoc time-varying inflation target given by one of the best core inflation measures, namely the PCE trimmed-mean core inflation. At the same time, we are evaluating the core inflation measure by directly incorporating them into a dynamic general equilibrium model. The analysis of the inflation dynamics, especially in correspondence to its broken-down components is interesting and worth exploring further. It is argued that the Fed has been actually targeting a time-varying inflation target consistent with the underlying inflation dynamics.

Analysis of New Keynesian Phillips Curve Relationship in An Estimated Nonlinear DSGE Model

This paper estimates a nonlinear DSGE model based on Amisano & Tristani (2010) with US data. The model is approximated up to the second order. Conditional particle filter is used to calculate the likelihood and Bayesian method is used to simulate the posterior distribution of the parameters. The analysis of the nonlinear NKPC better accommodates short-term sharp-turns of the dynamics in the economy. It shows

different impulse responses that are conditional on high or low inflation rate in the initial period. As a result, the relationships between inflation, its inertia, the expected inflation and output are more consistent with the theory suggested by the model.

Value at Risk (VaR) Based on GARCH-Type Estimated Volatility Models of 5 Stock Markets

The Great Recession has stirred up debate about risk management practices. Value-at-Risk (VaR) is often blamed for imprudent excessive risk taking leading to the crisis. VaR-based potential loss calculation is based on the assumption of normality of the shocks. In reality, shocks distributions are often highly kurtotic. VaR will be more accurately representing the real risks if such distributions are used in the calculation. Exponential GARCH(1,1) with Student's t-distribution is shown to be a more reliable model than the simple standard RiskMetrics and the standard GARCH(1,1) approaches in tracking the extreme values of the distribution of the daily log returns of the market indices.

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Contents

1	Empirical Analysis of A Core Inflation Measure in An Estimated DSGE Model	1
1.1	Introduction	1
1.2	The Model	4
1.3	The State Space Representation of the Model	12
1.4	The Data	17
1.5	Estimation	17
1.6	Results	17
1.7	Conclusion	19
2	Analysis of New Keynesian Phillips Curve Relationship in An Estimated Nonlinear DSGE Model	27
2.1	Introduction	27
2.2	Some empirical facts	29
2.3	The Model	31
2.3.1	Household	32
2.3.2	Firm	33
2.3.3	Taylor rule	35
2.4	Solving the Model	36
2.5	State space model and filtering	38
2.5.1	Particle Filter	40

2.5.2	Conditional Particle Filter	40
2.6	Data, Priors, and Simulation Time	41
2.7	Results	42
2.8	Conclusion	52
3	Value at Risk (VaR) Based on GARCH-Type Estimated Volatility Models of 5 Stock Markets	54
3.1	Introduction	54
3.2	Literature Review	56
3.3	Stylized Facts of Stock Market Daily Log Return Distributions	57
3.4	Methodology	61
3.5	Estimation	64
3.6	Risk model evaluation	65
3.6.1	Loss function	65
3.6.2	Backtest	66
3.7	Conclusion	69
A	Computer Codes	90
A.1	Parts of Matlab Code Used in Chapter 1	90
A.2	Parts of Matlab Code Used in Chapter 2	99
A.3	Parts of Stata Code Used in Chapter 3	111

List of Figures

1.1	Impulse Responses for Scenario 1: Output growth and Inflation	23
1.2	Impulse Responses for Scenario 1: Interest Rate and Output Gap	24
1.3	Impulse Responses for Scenario 2: Output growth and Inflation	25
1.4	Impulse Responses for Scenario 2: Interest Rate and Ouput Gap	26
2.1	US Inflation, Interest Rate and GDP Growth (Quarterly, 1972 - 2008)	30
2.2	Centered Moving Standard Deviations of GDP Growth and Inflation, and Lagged Short Term Correlation between Inflation and Output	31
2.3	Posterior and Prior Distributions of Parameters (1)	43
2.4	Posterior and Prior Distributions of Parameters (2)	44
2.6	Actual inflation, predicted inflation, and predicted inflation target	45
2.5	Actual data and fitted value for inflation, interest rate and output	45
2.7	Impulse responses: technology and inflation target shocks	49
2.8	Impulse responses: tax and monetary policy shocks	50
3.1	Daily Log Return of 5 Stock Markets, Aug-2000 to Jun-2012	59
3.2	Quantiles of Daily Log Returns Against Quantiles of Their Respective Associated Normal Distribution	60
3.3	1500 Days Moving Statistics of Daily Log Return of 5 Markets	62
3.4	AXKO: 2008 Out of sample forecast, RiskMetrics vs. EGARCH(1,1)-t	68

List of Tables

1.1	Estimation Results for both simulations	20
1.2	Forecast Error Variance Decompositions for Scenario 1 (PCE trimmed-mean as the inflation target)	21
1.3	Forecast Error Variance Decompositions for Scenario 2 (3% as the inflation target)	22
2.1	Results from Posterior Simulations	47
3.1	Summary Statistics of The Distribution of Daily Log Returns	60
3.2	Normality Tests	61
3.3	Model Rankings	67
3.4	AXKO: Backtest of Value at Risk of Mid-2011 to Mid-2012	70
3.5	HSI: Backtest of Value at Risk of Mid-2011 to Mid-2012	70
3.6	FTSE: Backtest of Value at Risk of Mid-2011 to Mid-2012	71
3.7	BVSP: Backtest of Value at Risk of Mid-2011 to Mid-2012	71
3.8	GSPC: Backtest of Value at Risk of Mid-2011 to Mid-2012	71
3.9	AXKO: Backtest of Value at Risk of 2008	72
3.10	HSI: Backtest of Value at Risk of 2008o	72
3.11	BVSP: Backtest of Value at Risk of 2008	72
3.12	FTSE: Backtest of Value at Risk of 2008	73
3.13	GSPC: Backtest of Value at Risk of 2008	73

3.14	AXKO: Volatility Model Estimation Results Using Daily Log Return, August 2000	
	- June 2011 (2737 trading days)	74
3.15	AXKO: Volatility Model Estimation Results Using Daily Log Return, August 2000	
	- June 2011 (2737 trading days)	75
3.16	HSI: Volatility Model Estimation Results Using Daily Log Return, August 2000 -	
	June 2011 (2714 trading days)	76
3.17	HSI: Volatility Model Estimation Results Using Daily Log Return, August 2000 -	
	June 2011 (2714 trading days)	77
3.18	FTSE: Volatility Model Estimation Results Using Daily Log Return, August 2000	
	- June 2011 (2749 trading days)	78
3.19	FTSE: Volatility Model Estimation Results Using Daily Log Return, August 2000	
	- June 2011 (2749 trading days)	79
3.20	BVSP: Volatility Model Estimation Results Using Daily Log Return, August 2000	
	- June 2011 (2687 trading days)	80
3.21	BVSP: Volatility Model Estimation Results Using Daily Log Return, August 2000	
	- June 2011 (2687 trading days)	81
3.22	GSPC: Volatility Model Estimation Results Using Daily Log Return, August 2000	
	- June 2011 (2739 trading days)	82
3.23	GSPC: Volatility Model Estimation Results Using Daily Log Return, August 2000	
	- June 2011 (2739 trading days)	83

Chapter 1

Empirical Analysis of A Core Inflation Measure in An Estimated DSGE Model

1.1 Introduction

The Fed does not explicitly target inflation. Nevertheless, along with targeting employment, it always reacts to change in expected inflation based on a Taylor rule type reaction function, which expresses the interest rate as a function of the output gap and the deviation between actual inflation with its target.

Since the early 1990s, along with the inflation targeting policy regime, core inflation has received much attention. The first country that adopted a fully pledged inflation targeting policy was New Zealand, whose central bank explicitly used CPI as their target. There have been many discussions about the topic of inflation targeting and core inflation since then. Due to relative benefits compared to the interest or the monetary aggregate targeting framework, some countries like UK, Spain, Korea, etc., followed suit to adopt an inflation targeting policy. For a review on the relative benefits of inflation targeting framework see, for example, Mishkin & Schmidt-Hebbel (2007).

Along with the inflation targeting, some discussion revolves around which price index to target. A central bank needs to answer this rather technical question because they want to be able to

effectively communicate their policy changes to the public. In some countries where the central bank is constitutionally responsible to the parliament, an explicit statement about which index or indicators to target serves as the yardstick to measure their success or failure.

There has been always a debate about the issue of which index to target. Some economists favor the CPI core inflation (CPI without the food and energy prices) while others prefer the headline CPI. Some economists even propose GDP deflator as the indicator. Like most inflation targeting countries, New Zealand started in 1990 with targeting the core inflation (which is the CPI excluding food and energy prices) but then changed it to the headline CPI in 2002. The Fed, which never really explicitly targets inflation, focuses on core personal consumption expenditure (PCE) deflator, which also excludes food and energy prices.

Given the vast variety of proposed measures, it is helpful to group them based on the methods used:

1. Measures based on exclusion. For example, CPI excluding the food and energy items; this is often coined as CPIX. This is the most popular measure and is usually what most central banks refer to as the core inflation measure.
2. Measures based on a long-run theoretical economic model. Core inflation is defined as that component of measured inflation that has no medium-to-long-run impact on real output as proposed by Quah & Vahey (1995). It is based on a VAR model consisting of price and output.
3. Measures based on stochastic approach, proposed by Roger (1997) and Bryan & Cecchetti (1999). It considers the price changes of goods and services as coming from a stochastic distribution of price changes. The problem of finding the core inflation is then reduced to a problem of finding the best central tendency of the distributions over time. The measures based on this approach belong to a class of trimmed-mean. Median can be seen as a 50% trimmed-mean (taking out 50% of the distribution from both sides) centered on the 50th

percentile. Mean, on the other hand, is the 0% trimmed-mean centered on the percentile of the mean itself.

In practice, there are at least three sources of data sets used to calculate core inflation: CPI, RPI and PCE deflator. How to choose the indicator and what criteria do we use? Generally there are two main criteria: the ability to track the inflation trend and the ability to forecast the inflation. Several economists have used both criteria and conclude that no particular core measure consistently outperform the others. See Hogan et al. (2001) and Rich & Steindel (2005) for examples.

However, there are some caveats to the first criteria. There is no guarantee that the measure of inflation trend itself is reliable. In practice, many use several months centered moving average as the trend indicator, while some use indicator based filters like HP filter or Band-Pass Filter. The introduction of these measures of so-called “trend” for inflation, to which the other indicators should be compared, is never “duly justified”. Robalo Marques et al. (2003) use a set of selection criteria based on its forecasting ability, although without the out of sample tests, and conclude that the trimmed-mean and the weighted median are both superior to the CPIX.

So far, these discussions fail to decide which measure provides the best indicator for the core inflation. Realistically, the central bank should not focus their attention only on a single measure. This is true even in the case that they believe one measure is better than the other. In this spirit we propose to directly use (and evaluate) the core inflation indicators in a dynamic general equilibrium model. The idea is that if a core inflation indicator is effective, it should have a sensible dynamic economic relationship with other macroeconomic variables in the model and improves the model fit in terms of its maximum likelihood. At this stage, we will use only the core inflation measure from the trimmed-mean class. In particular, we use PCE trimmed-mean calculated and published by Federal Reserve of Dallas. For our purposes, we will use one of the small scale versions of the New Keynesian DSGE model by Ireland (2004).

The Fed has the best knowledge of the underlying inflation. It is to its best interest to always understand if current inflation is in line with it. Following this logic, it makes sense for the Fed to

target the core inflation as the indicator for underlying inflation. This paper shows that if the Fed does this, the remaining inflation gap would be a good indicator for the supply shocks inflation that tend to be short-lived. We will show how this hypothetical scenario, compared to the standard practice of targeting a constant inflation target in modeling, will help in understanding more about the core inflation itself.

1.2 The Model

The model used below is heavily drawn from DeJong & Dave (2011). It is assumed that there are three different shocks to the economy: demand shock, preference shock and technology shock. There is a continuum of identical households. The representative household's problem is

$$\max_{c_t, m_t, n_t} U = E_0 \sum_{t=0}^{\infty} \beta^t \left[a_t \log c_t + \log \frac{m_t}{p_t} - \frac{n_t^\xi}{\xi} \right] \quad (1.1)$$

$$s.t. \quad pc_t + \frac{b_t}{r_t} + m_t = m_{t-1} + b_{t-1} + \tau_t + w_t n_t + d_t, \quad (1.2)$$

where b_t and m_t are the amount of bond and money the household holds respectively at time t . At maturity, the bond will have a gross nominal rate of return r_t . During each period, there is a lump sum transfer, τ_t from the central bank. The household receives wage, w_t for working for n_t hours. The household also owns an intermediate-goods firm which gives them dividend, d_t . The demand shock is denoted by a_t . The first order conditions of the household's optimization problem are given by

$$\left(\frac{w_t}{p_t} \right) \left(\frac{a_t}{c_t} \right) = n_t^{\xi-1} \quad (1.3)$$

$$\beta E_t \left[\left(\frac{1}{p_{t+1}} \right) \left(\frac{a_{t+1}}{c_{t+1}} \right) \right] = \left(\frac{1}{r_t p_t} \right) \left(\frac{a_t}{c_t} \right) \quad (1.4)$$

$$\left(\frac{m_t}{p_t}\right)^{-1} + \beta E_t \left[\left(\frac{1}{p_{t+1}}\right) \left(\frac{a_{t+1}}{c_{t+1}}\right) \right] = \left(\frac{1}{p_t}\right) \left(\frac{a_t}{c_t}\right) \quad (1.5)$$

There are two types of firms in this economy: the final consumption goods firm and the intermediate-goods firms. The final-good firm's problem is

$$\max_{y_{it}} \Pi_t^F = p_t y_t - \int_0^1 p_{it} y_{it} di \quad (1.6)$$

$$s.t. \quad y_t = \left[\int_0^1 y_{it}^{\left(\frac{\theta_t-1}{\theta_t}\right)} di \right]^{\left(\frac{\theta_t}{\theta_t-1}\right)} \quad (1.7)$$

The solutions to this problem are given by

$$y_{it} = y_t \left[\frac{p_{it}}{p_t} \right]^{-\theta_t} \quad (1.8)$$

$$p_t = \left[\int_0^1 p_{it}^{1-\theta_t} di \right]^{\left(\frac{1}{1-\theta_t}\right)}. \quad (1.9)$$

On the other hand, the representative intermediate-good firm's problem is

$$\max_{p_{it}} \Pi_{it}^I = E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{a_t}{c_t}\right) \left(\frac{d_t}{p_t}\right) \quad (1.10)$$

$$s.t. \quad y_{it} = z_t n_{it} \quad (1.11)$$

$$y_{it} = y_t \left[\frac{p_{it}}{p_t} \right]^{-\theta_t}. \quad (1.12)$$

$$\chi(p_{it}, p_{it-1}) = \frac{\phi}{2} \left[\frac{p_{it}}{\bar{\pi} p_{it-1}} - 1 \right]^2 y_t, \quad \phi > 0, \quad (1.13)$$

where

$$\frac{d_t}{p_t} = \frac{p_{it}y_{it} - w_t n_{it}}{p_t} - \chi(p_{it}, p_{it-1}). \quad (1.14)$$

The first order condition of this problem is given by

$$\begin{aligned} & (\theta_t - 1) \left(\frac{p_{it}}{p_t} \right)^{-\theta_t} \frac{y_t}{p_t} = \\ & \theta_t \left(\frac{p_{it}}{p_t} \right)^{-\theta_t-1} \frac{w_t}{p_t} \frac{y_t}{z_t} \frac{1}{p_t} - \left\{ \phi \left[\frac{p_{it}}{\bar{\pi} p_{it-1}} - 1 \right] \frac{y_t}{\bar{\pi} p_{it-1}} - \beta \phi E_t \left(\frac{a_{t+1}}{a_t} \frac{c_t}{c_{t+1}} \left(\frac{p_{it+1}}{\bar{\pi} p_{it}} - 1 \right) \frac{y_{t+1} p_{it+1}}{\bar{\pi} p_{it}^2} \right) \right\}. \end{aligned} \quad (1.15)$$

The left and right hand sides respectively are the marginal benefit and the marginal cost of increasing the relative price.

The central bank decides the nominal interest rate by following a Taylor Rule,

$$\begin{aligned} \tilde{r}_t &= \rho_r \tilde{r}_{t-1} + \rho_\pi \tilde{\pi}_t + \rho_g \tilde{g}_t + \rho_o \tilde{o}_t + \varepsilon_{rt} \\ \varepsilon_{rt} &\sim iid N(0, \sigma_r^2), \end{aligned} \quad (1.16)$$

where \tilde{r}_t , $\tilde{\pi}_t$, \tilde{g}_t , \tilde{o}_t are the nominal interest rate, the gross inflation rate, the gross growth rate of output and the output gap respectively. Each is in terms of log deviation from its own steady state value.

The output gap, \tilde{o}_t is the log difference of the output, y_t with its full employment level, \hat{y}_t based on a benevolent social planner's problem,

$$\max_{\hat{y}_t, n_{it}} U^S = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ a_t \log \hat{y}_t - \frac{1}{\xi} \left(\int_0^1 n_{it} di \right)^\xi \right\} \quad (1.17)$$

$$s.t. \quad \hat{y}_t = z_t \left(\int_0^1 n_{it}^{\left(\frac{\theta_t-1}{\theta_t} \right)} di \right)^{\left(\frac{\theta_t}{\theta_t-1} \right)}, \quad (1.18)$$

which symmetric solution is given by

$$\hat{y}_t = a_t^{\left[\frac{1}{\xi} \right]} z_t. \quad (1.19)$$

The demand shock , technology shock , and the cost-push shock are assumed to follow

$$\log a_t = (1 - \rho_a) \log \bar{a} + \rho_a \log a_{t-1} + \varepsilon_{at}, \quad (1.20)$$

$$\log z_t = \log \bar{z} + \log z_{t-1} + \varepsilon_{zt}, \quad (1.21)$$

$$\log \theta_t = (1 - \rho_\theta) \log \bar{\theta} + \rho_\theta \log \theta_{t-1} + \varepsilon_{\theta t} \quad (1.22)$$

where $(\bar{a}, \bar{z}, \bar{\theta}) > 1$ and $|\rho_i| < 1$, $i = \{a, z, \theta\}$.

To close the model, symmetry is assumed on the intermediate-goods firms. This implies

$$y_{it} = y_t, \quad n_{it} = n_t, \quad p_{it} = p_t, \quad d_{it} = d_t \quad (1.23)$$

Also, the money and bond markets are assumed to be at equilibrium,

$$m_t = m_{t-1} + \tau_t, \quad (1.24)$$

$$b = b_{t-1} = 0 \quad (1.25)$$

So far, the model involves twelve equations: the household's budget constraint and first order conditions, the economy's production function, the aggregate real dividends claimed by the households from the intermediate-goods firm, the first order condition of the intermediate-goods firm's optimization problem, structural shocks equations, and the potential output statement.

As in Ireland (2004), we want to reduce this into a linearized eight-equation system consisting of an IS curve, a Phillips curve, a Taylor Rule specification, the three exogenous shock specifications, and definitions for both the growth rate of output and the output gap. To get to the reduced system we need to normalize some variables. The normalized variables are

$$\ddot{y}_t = \frac{y_t}{z_t}, \quad \ddot{c}_t = \frac{c_t}{z_t}, \quad \ddot{\hat{y}}_t = \frac{\hat{y}_t}{z_t}, \quad \pi_t = \frac{p_t}{p_{t-1}},$$

$$\ddot{d}_t = \frac{(d_t/p_t)}{z_t}, \quad \ddot{w}_t = \frac{(w_t/p_t)}{z_t}, \quad \ddot{m}_t = \frac{(m_t/p_t)}{z_t}, \quad \ddot{z}_t = \frac{z_t}{z_{t-1}}.$$

Using the expression for the real dividend from equation (1.14), the household's budget constraint in equation (1.2) can be transformed as

$$\ddot{y}_t = \ddot{c}_t + \frac{\phi}{2} \left[\frac{\pi_t}{\bar{\pi}} - 1 \right]^2 \ddot{y}_t \quad (1.26)$$

On the other hand, the household's first-order condition (1.4) can be normalized as

$$\frac{a_t}{\ddot{c}_t} = \beta r_t E_t \left[\frac{a_{t+1}}{\ddot{c}_{t+1}} \times \frac{1}{\ddot{z}_{t+1}} \times \frac{1}{\pi_{t+1}} \right]. \quad (1.27)$$

The household's remaining first-order conditions, the real dividend payment equation, and the aggregate production function can be used to substitute out wages, money, labor, dividends, and capacity output from the system. And then, we express the output gap as

$$o_t \equiv \frac{y_t}{\hat{y}_t} = \frac{\ddot{y}_t}{a_t \left[\frac{1}{\xi} \right]}. \quad (1.28)$$

After normalizing the first-order condition of the intermediate-goods firm and the stochastic specification for the technology shock, we are left with a system of nonlinear equations (1.29) – (1.36).

$$\ddot{y}_t = \ddot{c}_t + \frac{\phi}{2} \left(\frac{\pi_t}{\bar{\pi}} - 1 \right)^2 \ddot{y}_t \quad (1.29)$$

$$\frac{a_t}{\ddot{c}_t} = \beta r_t E_t \left[\frac{a_{t+1}}{\ddot{c}_{t+1}} \times \frac{1}{\ddot{z}_{t+1}} \times \frac{1}{\pi_{t+1}} \right] \quad (1.30)$$

$$0 = 1 - \theta_t + \theta_t \frac{\ddot{c}_t}{a_t} \ddot{y}_t^{\xi-1} - \phi \left(\frac{\pi_t}{\bar{\pi}} - 1 \right) \frac{\pi_t}{\bar{\pi}} + \beta \phi E_t \quad (1.31)$$

$$g_t = \frac{\ddot{z}_t \ddot{y}_t}{\ddot{y}_{t-1}} \quad (1.32)$$

$$o_t \equiv \frac{y_t}{\hat{y}_t} = \frac{\ddot{y}_t}{a_t^{\left[\frac{1}{\xi}\right]}} \quad (1.33)$$

$$\log a_t = (1 - \rho_a) \log \bar{a} + \rho_a \log a_{t-1} + \varepsilon_{at} \quad (1.34)$$

$$\log \theta_t = (1 - \rho_\theta) \log \bar{\theta} + \rho_\theta \log \theta_{t-1} + \varepsilon_{\theta t} \quad (1.35)$$

$$\log \ddot{z}_t = \log \bar{z} + \varepsilon_{zt} \quad (1.36)$$

Along with the Taylor Rule, we will linearize this system. The log-linearization process begins with the calculation of the steady state values for all endogenous variables of the system which are

$$\begin{aligned} \bar{r} &= \frac{\bar{z}}{\bar{\beta}} \bar{\pi}, \\ \bar{c} = \bar{y} &= \bar{a} \frac{\bar{\theta}-1}{\bar{\theta}}^{\left[\frac{1}{\xi}\right]} \\ \bar{o} &= \left(\frac{\bar{\theta}-1}{\bar{\theta}} \right)^{\frac{1}{\xi}}. \end{aligned} \quad (1.37)$$

First, log-linearizing equation (1.29) yields $\tilde{y}_t \equiv \log \left(\frac{\ddot{y}_t}{\bar{y}} \right) = \tilde{c}_t$, where the tilde denotes the logged deviation of a variable from its steady state value. Hence, this equation can be removed from the system and \tilde{y}_t will substitute \tilde{c}_t throughout the system. Next, log-linearizing equation (1.30) will yield

$$0 = \tilde{r}_t - E_t \tilde{\pi}_{t+1} - (E_t \tilde{y}_{t+1} - \tilde{y}_t) + E_t \tilde{y}_{t+1} - \tilde{a}_t. \quad (1.38)$$

While log-linearizing the output gap equation (1.33) yields

$$\tilde{y}_t = \frac{1}{\xi} \tilde{a}_t + \tilde{o}_t. \quad (1.39)$$

Now, we can use the expression for the output gap from equation (1.39) to substitute out the term in equation (1.38). By doing so we will get the expression for the IS curve,

$$\tilde{o}_t = E_t \tilde{o}_{t+1} - (\tilde{r}_t - E_t \tilde{\pi}_{t+1}) + (1 - \xi^{-1}) (1 - \rho_a) \tilde{a}_t. \quad (1.40)$$

On the other hand, log-linearizing equation (1.31), \tilde{y}_t from equation (1.39) will yield the Phillips curve,

$$\tilde{\pi}_t = \beta E_t \tilde{\pi}_{t+1} + \psi \tilde{o}_t - \tilde{e}_t, \quad (1.41)$$

where $\psi = \frac{\xi(\theta-1)}{\phi}$ and $\tilde{e}_t = \frac{1}{\phi} \tilde{\theta}_t$.

As the final step, the Phillips curve and the IS curve are enhanced by adding the lagged variables of the output gap and the inflation. Denoting \tilde{o}_{t-1} as \tilde{o}^- , \tilde{o}_t as \tilde{o} , \tilde{o}_{t+1} as \tilde{o}' , and so on, we can then drop the time subscript and write the log-linearized system.

$$\tilde{o} = \alpha_o \tilde{o}^- + (1 - \alpha_o) E_t \tilde{o}' - (\tilde{r} - E_t \tilde{\pi}') + (1 - \omega) (1 - \rho_a) \tilde{a} \quad (1.42)$$

$$\tilde{\pi} = \beta \alpha_\pi \tilde{\pi}^- + \beta (1 - \alpha_\pi) E_t \tilde{\pi}' + \psi \tilde{o} - \tilde{e} \quad (1.43)$$

$$\tilde{g}' = \tilde{y}' - \tilde{y} + \tilde{z}' \quad (1.44)$$

$$\tilde{o}' = \tilde{y}' - \omega \tilde{a}' \quad (1.45)$$

$$\tilde{r}' = \tilde{r} + \rho_\pi \tilde{\pi}' + \rho_g \tilde{g}' + \rho_o \tilde{o}' + \varepsilon_r' \quad (1.46)$$

$$\tilde{a}' = \rho_a \tilde{a} + \varepsilon_a' \quad (1.47)$$

$$\tilde{e}' = \rho_e \tilde{e} + \varepsilon_e' \quad (1.48)$$

$$\tilde{z}' = \varepsilon_z' \quad (1.49)$$

Where the structural shocks $v_t = \{\varepsilon_{rt}, \varepsilon_{at}, \varepsilon_{et}, \varepsilon_{zt}\}$ are *iidN* with diagonal covariance matrix Σ , $\alpha_o \in [0, 1]$, and $\alpha_\pi \in [0, 1]$. Note that a new parameter ω in and has been defined as $\omega = 1/\xi$.

For the purpose of the estimation of the linearized model, it is first necessary to map this system into the first-order specification. Equations (1.42) - (1.46) can be written in the form of

$$AE_t x_{t+1}^0 = Bx_t^0 + Cv_t. \quad (1.50)$$

Since the IS curve and Phillips curve are augmented to include the lagged values of the output gap and inflation, we need to augment the vector x_t to also include these lagged values. Consequently, $x_t^0 \equiv [\tilde{y}^- \tilde{r}^- \tilde{\pi}^- \tilde{g}^- \tilde{o}^- \tilde{\pi} \tilde{o}]'$. On the other hand, $v_t = [\tilde{a} \tilde{e} \tilde{z} \varepsilon_r]'$.

The corresponding matrices A , B , and C are then given by

$$A = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 1 & 1 - \alpha_o \\ 0 & 0 & 0 & 0 & \psi & \beta(1 - \alpha_\pi) & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -\rho_\pi & -\rho_g & -\rho_o & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \quad (1.51)$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & -\alpha_o & 0 & 1 \\ 0 & 0 & -\beta\alpha_\pi & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad (1.52)$$

and

$$C = \begin{bmatrix} 0 - (1 - \omega)(1 - \rho_a) & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \omega & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \quad (1.53)$$

Meanwhile, equations (1.47) through (1.49), can be written as

$$v_t = P v_{t-1} + \varepsilon_t, \quad (1.54)$$

where

$$P = \begin{bmatrix} \rho_a & 0 & 0 & 0 \\ 0 & \rho_e & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \text{ and } \varepsilon_t = [\varepsilon_{at} \varepsilon_{et} \varepsilon_{zt} \varepsilon_{rt}]'.$$

Now, we need to identify the observable variables, which are the gross growth rate of output g_t , the gross inflation rate π_t , and the nominal interest rate r_t (all are as their logged ratios of sample averages). Assuming that all g_t , π_t , and r_t are stationary, the analysis can proceed to solve the linearized system (1.42) - (1.49).

1.3 The State Space Representation of the Model

Equations (1.42) through (1.49) form a system with three observable variables (output growth \tilde{g}_t , inflation $\tilde{\pi}_t$, and the short-term nominal interest rate \tilde{r}_t), two unobservable variables (stochastically detrended output \tilde{y}_t , and the output gap \tilde{o}_t), and four unobservable shocks (to preferences ε_{at} , desired markups ε_{et} , technology ε_{zt} , and monetary policy ε_{rt}). The solution to this system,

derived using Klein (2000) procedure, takes the form of a state-space econometric model. Hence, a Kalman filtering algorithms outlined by Hamilton (1994) can be applied to estimate the model's parameters via maximum likelihood and to draw inferences about the behavior of the model's unobservable components based on the information contained in the three observable series.

To get the state space model, we need to solve the system of (1.50), which is a system of linear expectational difference equations. First, we decompose A and B by using QZ factorization into unitary upper triangular matrices

$$A = Q' \Lambda Z' \quad (1.55)$$

$$B = Q' \Omega Z', \quad (1.56)$$

where (Q, Z) are unitary¹ and (Λ, Ω) are upper triangular. Next, we order (Q, Z, Λ, Ω) in such a way that, in absolute value, the generalized eigenvalues of A and B are organized in Λ and Ω in increasing order moving from left to right. The generalized eigenvalues of Θ are obtained as the solution to $\Theta e = \lambda \Xi e$, where Ξ is asymmetric matrix.

In this particular system, we have five predetermined variables and two non-predetermined variables in the vector x_t . Hence, to have a unique solution, we need five of the generalized eigenvalues lie inside the unit circle and two of the generalized eigenvalues lie outside the unit circle².

Assume that we have exactly two generalized eigenvalues that lie outside the unit circle, then we can partition the matrices (Q, Z, Λ, Ω) conformably such that

¹ A unitary matrix Θ satisfies $\Theta' \Theta = I$.

² If more than (less than) two of the generalized eigenvalues lie outside the unit circle, then the system has no solution (multiple solutions).

$$Q = \begin{bmatrix} Q_{1(5 \times 7)} \\ Q_{1(2 \times 7)} \end{bmatrix}, \quad Z = \begin{bmatrix} Z_{11(5 \times 5)} & Z_{12(5 \times 2)} \\ Z_{21(2 \times 5)} & Z_{22(2 \times 2)} \end{bmatrix}, \quad \Lambda = \begin{bmatrix} \Lambda_{11(5 \times 5)} & \Lambda_{12(5 \times 2)} \\ 0_{(2 \times 5)} & \Lambda_{22(2 \times 2)} \end{bmatrix}, \quad \text{and } \Omega = \begin{bmatrix} \Omega_{11(5 \times 5)} & \Omega_{12(5 \times 2)} \\ 0_{(2 \times 5)} & \Omega_{22(2 \times 2)} \end{bmatrix}.$$

Next, let $x_t^1 = Z' x_t^0$ such that, in particular,

$$x_t^1 = \begin{bmatrix} x_{1t}^1 \\ x_{2t}^1 \end{bmatrix},$$

where

$$x_{1t(5 \times 1)}^1 = Z'_{11} \begin{bmatrix} \tilde{y}^{-1} \\ \tilde{r}^{-1} \\ \tilde{\pi}^{-1} \\ \tilde{g}^{-1} \\ \tilde{o}^{-1} \end{bmatrix} + Z'_{21} \begin{bmatrix} \tilde{\pi} \\ \tilde{o} \end{bmatrix} \quad (1.57)$$

and

$$x_{2t(2 \times 1)}^1 = Z'_{12} \begin{bmatrix} \tilde{y}^{-1} \\ \tilde{r}^{-1} \\ \tilde{\pi}^{-1} \\ \tilde{g}^{-1} \\ \tilde{o}^{-1} \end{bmatrix} + Z'_{22} \begin{bmatrix} \tilde{\pi} \\ \tilde{o} \end{bmatrix} \quad (1.58)$$

Premultiplying (1.50) by Q , using the fact that $x_t^1 = Z' x_t^0$ and that Z is unitary, will yield

$$\Lambda E_t x_{t+1}^1 = \Omega x_t^1 + Q C v_t$$

or, in the form of matrix partitions,

$$\Lambda_{11} E_t x_{1t+1}^1 + \Lambda_{12} E_t x_{2t+1}^1 = \Omega_{11} x_{1t}^1 + \Omega_{12} x_{2t}^1 + Q_1 C v_t \quad (1.59)$$

and

$$\Lambda_{22}E_t x_{2t+1}^1 = \Omega_{22}x_{2t}^1 + Q_2 C v_t \quad (1.60)$$

Because the generalized eigenvalues of each of Λ_{22} and Ω_{22} all lie outside the unit circle, (1.60) can be solved as where H is a 2×4 matrix given by

$$x_{2t}^1 = -\Omega_{22}^{-1} H v_t, \quad (1.61)$$

where H is a 2×4 matrix given by

$$\begin{aligned} \text{vec}(R) &= \text{vec} \sum_{j=0}^{\infty} (\Lambda_{22} \Omega_{22}^{-1})^j Q_2 C P^j \\ &= \sum_{j=0}^{\infty} \text{vec} \left[(\Lambda_{22} \Omega_{22}^{-1})^j Q_2 C P^j \right] \\ &= \sum_{j=0}^{\infty} \left[P^j \otimes (\Lambda_{22} \Omega_{22}^{-1})^j \right] \text{vec}(Q_2 C) \\ &= \sum_{j=0}^{\infty} \left[P \otimes (\Lambda_{22} \Omega_{22}^{-1}) \right]^j \text{vec}(Q_2 C) \\ &= \left[I_{(8 \times 8)} - P \otimes (\Lambda_{22} \Omega_{22}^{-1}) \right]^{-1} \text{vec}(Q_2 C). \end{aligned}$$

Now, we can use (1.58) and (1.61) to solve for

$$\begin{bmatrix} \tilde{\pi} \\ \tilde{o} \end{bmatrix} = - \left(Z'_{22} \right)^{-1} Z'_{12} \begin{bmatrix} \tilde{y}^{-1} \\ \tilde{r}^{-1} \\ \tilde{\pi}^{-1} \\ \tilde{g}^{-1} \\ \tilde{o}^{-1} \end{bmatrix} - \left(Z'_{22} \right)^{-1} \Omega_{22}^{-1} H v. \quad (1.62)$$

Using the fact that Z is unitary, we can write

$$\left(Z'_{22} \right)^{-1} = Z_{22} - Z_{21} Z_{11}^{-1} Z_{12}.$$

This allows us to rewrite (1.62) as

$$\begin{bmatrix} \tilde{\pi} \\ \tilde{o} \end{bmatrix} = M_1 \begin{bmatrix} \tilde{y}^{-1} \\ \tilde{r}^{-1} \\ \tilde{\pi}^{-1} \\ \tilde{g}^{-1} \\ \tilde{o}^{-1} \end{bmatrix} + M_2 v, \quad (1.63)$$

where $M_1 = Z_{21}Z_{11}^{-1}$, and $M_2 = -[Z_{22} - Z_{21}Z_{11}^{-1}Z_{12}]\Omega_{22}^{-1}H$.

By doing the same exercise, we can rewrite (1.57) as

$$x_{1t}^1 = Z_{11}' \begin{bmatrix} \tilde{y}^{-1} \\ \tilde{r}^{-1} \\ \tilde{\pi}^{-1} \\ \tilde{g}^{-1} \\ \tilde{o}^{-1} \end{bmatrix} + Z_{11}^{-1}Z_{12}\Omega_{22}^{-1}Hv_t. \quad (1.64)$$

Substituting (1.64) into (1.59) will yield

$$\begin{bmatrix} \tilde{y} \\ \tilde{r} \\ \tilde{\pi} \\ \tilde{g} \\ \tilde{o} \end{bmatrix} = M_3 \begin{bmatrix} \tilde{y}^{-1} \\ \tilde{r}^{-1} \\ \tilde{\pi}^{-1} \\ \tilde{g}^{-1} \\ \tilde{o}^{-1} \end{bmatrix} + M_4 v, \quad (1.65)$$

where $M_3 = Z_{11}\Lambda_{11}^{-1}\Omega_{11}Z_{11}^{-1}$ and $M_4 = Z_{11}\Lambda_{11}^{-1}(\Omega_{11}Z_{11}^{-1}Z_{12}\Omega_{22}^{-1}H + Q_1C + \Lambda_{12}\Omega_{22}^{-1}HP - \Omega_{12}\Omega_{22}^{-1}H) - Z_{12}\Omega_{22}^{-1}HP$.

Now, equations (1.65) and (1.54) form a state space model that can be analyzed by using a Kalman filter. We can rewrite the two equations in a more compact form as

$$x_{t+1} = \Pi x_t + W\varepsilon_{t+1}, \quad (1.66)$$

where $x_t = [\tilde{y}^{-1} \tilde{r}^{-1} \tilde{\pi}^{-1} \tilde{g}^{-1} \tilde{o}^{-1} \tilde{a} \tilde{e} \tilde{z} \varepsilon_r]'$, $\varepsilon_{t+1} = [\varepsilon'_d \varepsilon'_e \varepsilon'_z \varepsilon'_r]'$, $\Pi = \begin{bmatrix} M_3 & M_4 \\ 0_{(4 \times 5)} & P \end{bmatrix}$, and $W = \begin{bmatrix} 0_{5 \times 4} \\ I_{4 \times 4} \end{bmatrix}$.

1.4 The Data

The data comprises the quarterly output growth, inflation, and nominal interest rate covering the period of 1948:I through 2009:IV. The output growth calculated as quarterly changes of seasonally adjusted real GDP, normalized to per capita terms by dividing it by the civilian non-institutional population aged 16 and over. The inflation measure is calculated as the quarterly changes of the seasonally adjusted GDP deflator. While the nominal interest rate is the quarterly averages of daily 3-month US Treasury bill rate.

1.5 Estimation

The Kalman filter is used to estimate the maximum likelihood function based on a state space model. The observable variables are output growth, inflation rate, and interest rate. We run two simulations as follows: (1) Core inflation (PCE trimmed-mean) as the inflation target and (2) 3%/year as the inflation target.

In both scenarios, we follow econometric strategies adopted by Ireland (2004). Most notably, we fix the slope of the Phillips curve, $\psi=0.1$. Ireland (2004) explains that based on Gali & Gertler (1999), in a simpler of the New Keynesian model with Calvo (1983) contract specification, a value of 0.1 corresponds to the case where individual goods prices are reset on average every 3.74 quarters.

1.6 Results

As Table 1.1 shows, the likelihood is higher for the first scenario. This is in a way confirming that, over the sample period, the Fed has adopted a more time-varying inflation target consistent with the core inflation measure instead of a fixed one. Bernanke et al. (2001) discusses why it is important for an inflation targeting central bank to announce its target regularly to public with their

justifying rationale. This is important to do to especially after a period of low and stable inflation to avoid misunderstanding by the public to assume that the inflation target is a fixed one.

The implied labor inelasticity is suggested to be really high by the model for both scenarios. This is indicated by a very small estimated value for ω . Nevertheless, given the fact that ψ is set to equal to 0.1 to begin with, as in Ireland (2004), this simply means that the data like the model where the efficient level of output is basically unaffected by the preference shock.

The output gap persistence in the IS curve turns out to be high in the first scenario, $\alpha_o = 0.9993$. The estimated backward-looking inflation coefficient in the Phillips curve, $\alpha_\pi = 0$ for both scenarios. This is indicating that in both scenarios, the inflation is purely forward-looking.

Taylor rule coefficients for output growth, ρ_g are high in both scenarios. Though still quite high, in the first scenario, it is less than half of the parameter value in the second scenario. Similarly, the coefficient for the output gap, ρ_o in the first scenario is also less than half of the parameter value in the second scenario. This makes sense because when the Fed is targeting the core inflation, its monetary policy will respond to output's dynamic less frequently. After all, the core inflation is assumed to be caused by aggregate demand and inflation expectation dynamics. This means that pure supply shocks, which are usually short-lived, were mostly recognized by the Fed during the sample period, to which the Fed should not respond by changing interest rate. On the other hand, the coefficient for expected inflation, ρ_π is similar between the two scenarios. Again, this is confirming that core inflation measure captures the underlying component of the inflation quite well.

The parameter for cost-push shock persistence gives us a much clearer distinction between the two scenarios. ρ_θ is 0.00 for the first scenario and 0.9898 for the second scenario. In the first scenario, there are still cost push shocks realized and acknowledged. Nevertheless, they should be short-lived. The feed to change of expected inflation still exists, especially if the Fed lowers the interest rate to accommodate the negative output gap the supply shock usually brings about.

The estimates for the standard deviations of the shocks, σ_a , σ_θ , σ_z , and σ_r are all smaller but one for the first scenario. Statistically speaking, this is to be expected. After all, the core inflation

series is clearly much less volatile than the actual measured inflation series. Interestingly, the shock that is larger for the first scenario compared to the second one is the cost-push shock. This indicates that the core inflation measure has inherently absorbed much of the volatilities in the inflation dynamic. Nevertheless, it does not absorb the cost-push shock. In fact, it accentuates the cost-push components of the inflation dynamic.

The very same message can be deduced from the forecast error variance decomposition in Table 1.2. Especially on the inflation dynamic in the first scenario, clearly the cost-push shock is the major determinant in short term and is decreasing in role in the long run. In contrast, when the inflation target is assumed to be fixed at 3%, Table 1.3 gives us the opposite story where cost-push shock is not the main determinant of inflation in the short run but in the long run.

The impulse responses tell us similar story. In scenario one, a one standard deviation cost-push shock changes the inflation by 0.6 percentage point. Nevertheless, this effect quickly disappears after 2 quarters. The opposite is true in scenario two. The same magnitude of cost-push shock causes the inflation to change by about 0.5 percentage point but the effect lingers even after 20 quarters. This is not a quite theoretically consistent scenario in this regard. The impulse response functions for other variables are mostly theoretically sensible, especially for the first scenario related to cost-push innovation.

Output growth, output gap, and interest are almost unaffected by cost-push shock. On the other hand, a monetary policy shock affects the output growth temporarily and the effect disappears after 4 quarters. The same monetary policy shock affects the inflation negatively and disappears after about 8 quarters. The interest rate would actually slightly negatively affected initially and then reversed after 2 quarters. This is due to the fact that the Phillips curve is highly forward-looking.

1.7 Conclusion

One of the best core inflation measure, i.e. PCE trimmed-mean is used as the time-varying inflation target in a simple estimated New Keynesian Ireland (2004) DSGE model. The Fed is assumed to follow this time-varying inflation target instead of a fixed one as usually done in a

Table 1.1: Estimation Results for both simulations

Parameters	Model 1	Model 2
ω	0.0021	0.0189
α_o	0.9993	0.0718
α_π	0.0000	0.0000
ρ_π	0.4746	0.4973
ρ_g	1.3136	3.9498
ρ_o	0.2048	0.4512
ρ_a	0.9702	0.9878
ρ_θ	0.0000	0.9898
σ_a	0.2644	0.7294
σ_θ	0.0072	0.0004
σ_z	0.0087	0.0149
σ_r	0.0107	0.0285
Likelihood Value	1362.02	1310.08

standard DSGE modeling. Although not the main point of this study, as the result, the model does a better job in tracking the data.

The analysis of the inflation dynamics, especially in correspondence to its broken-down components is interesting and worth exploring further. It is argued that the Fed has been actually targeting a time-varying inflation target consistent with the underlying inflation dynamics. The cost-push shock is shown to be dominant in the short run and tends to be short-lived. Consequently, the demand and expected inflation play dominant role in medium to long run.

Table 1.2: Forecast Error Variance Decompositions for Scenario 1 (PCE trimmed-mean as the inflation target)

Quarters ahead	Preference Shock	Cost-Push Shock	Technology Shock	Policy Shock
	Output Growth			
1	25.32	4.37	27.83	42.49
4	24.42	4.31	29.33	41.94
8	23.72	4.26	30.57	41.45
12	23.68	4.26	30.65	41.41
20	23.68	4.26	30.65	41.41
40	23.69	4.26	30.65	41.4
100	23.69	4.26	30.65	41.4
	Inflation			
1	3.49	69.73	14.29	12.49
4	5.1	50.72	23.58	20.61
8	5.01	48.99	24.55	21.45
12	5.22	48.82	24.53	21.44
20	5.7	48.56	24.4	21.33
40	6.28	48.27	24.25	21.2
100	6.52	48.14	24.19	21.14
	Interest Rate			
1	73.83	1.2	13.32	11.65
4	88.43	0.53	5.89	5.15
8	91.14	0.41	4.51	3.94
12	92.98	0.32	3.57	3.12
20	94.57	0.25	2.77	2.42
40	95.66	0.2	2.21	1.93
100	96	0.18	2.03	1.78
	Output Gap			
1	15.85	3.85	42.85	37.45
4	15.57	3.86	42.99	37.58
8	15.35	3.87	43.11	37.68
12	15.3	3.87	43.13	37.7
20	15.31	3.87	43.13	37.69
40	15.33	3.87	43.11	37.68
100	15.33	3.87	43.11	37.68

Table 1.3: Forecast Error Variance Decompositions for Scenario 2 (3% as the inflation target)

Quarters ahead	Preference Shock	Cost-Push Shock	Technology Shock	Policy Shock
	Output Growth			
1	22.26	0.23	4.48	73.03
4	23.38	1.27	17.07	58.28
8	23.38	1.51	20.79	54.32
12	23.24	1.54	21.59	53.63
20	23.16	1.54	21.83	53.47
40	23.36	1.55	21.78	53.31
100	23.76	1.58	21.65	53.01
	Inflation			
1	20.57	21.26	47.18	10.99
4	18.17	26.09	45.21	10.53
8	15.76	31.92	42.44	9.89
12	14.28	36.53	39.89	9.29
20	12.97	42.79	35.88	8.36
40	12.09	50.74	30.14	7.02
100	10.89	60.73	23.02	5.36
	Interest Rate			
1	59.41	21.4	15.56	3.63
4	63.54	19.43	13.81	3.22
8	72.19	16.81	8.92	2.08
12	77.22	15	6.31	1.47
20	82.06	13.03	3.99	0.93
40	85.6	11.51	2.35	0.55
100	86.85	11.41	1.41	0.33
	Output Gap			
1	32.36	0.04	54.82	12.77
4	32.09	0.27	54.86	12.78
8	31.45	1.67	54.24	12.64
12	30.61	3.94	53.08	12.37
20	28.89	9.08	50.31	11.72
40	25.85	18.65	45.02	10.49
100	25.85	18.65	45.02	10.49

Figure 1.1: Impulse Responses for Scenario 1: Output growth and Inflation

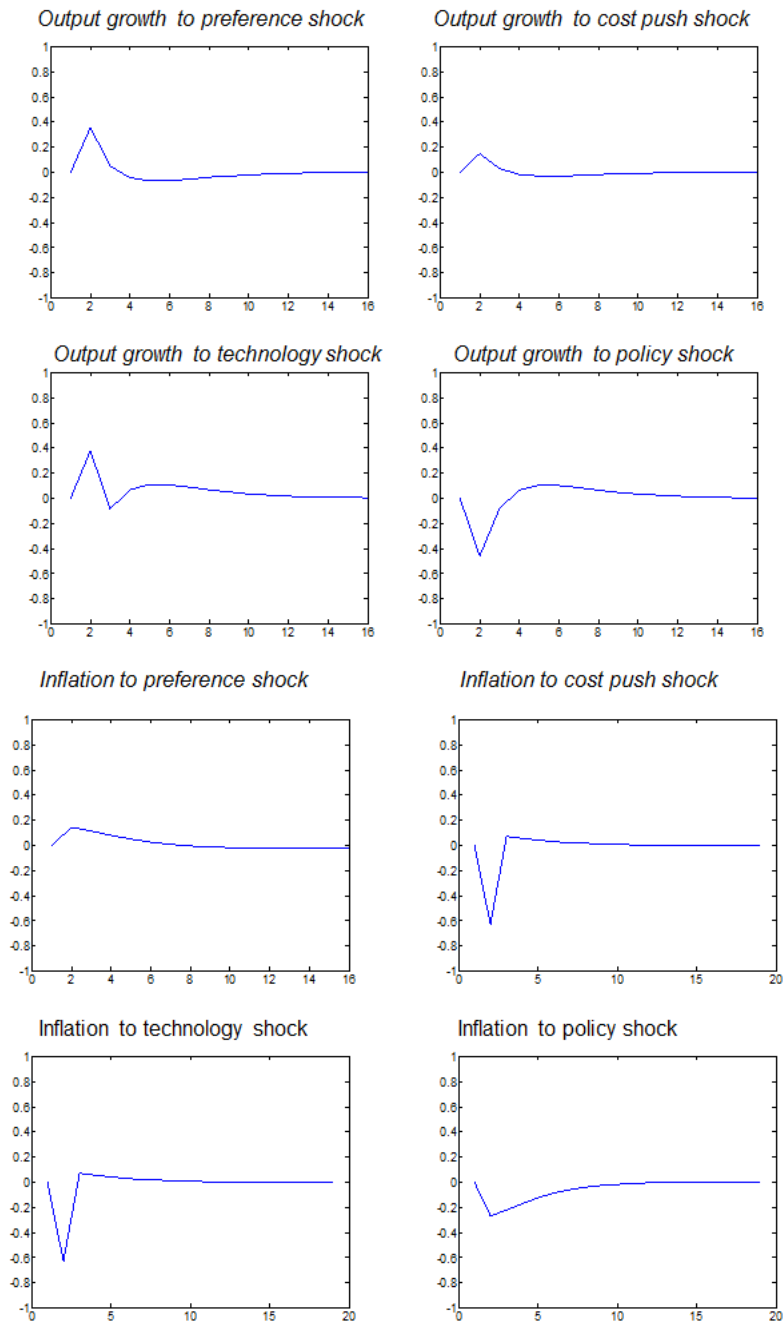


Figure 1.2: Impulse Responses for Scenario 1: Interest Rate and Output Gap

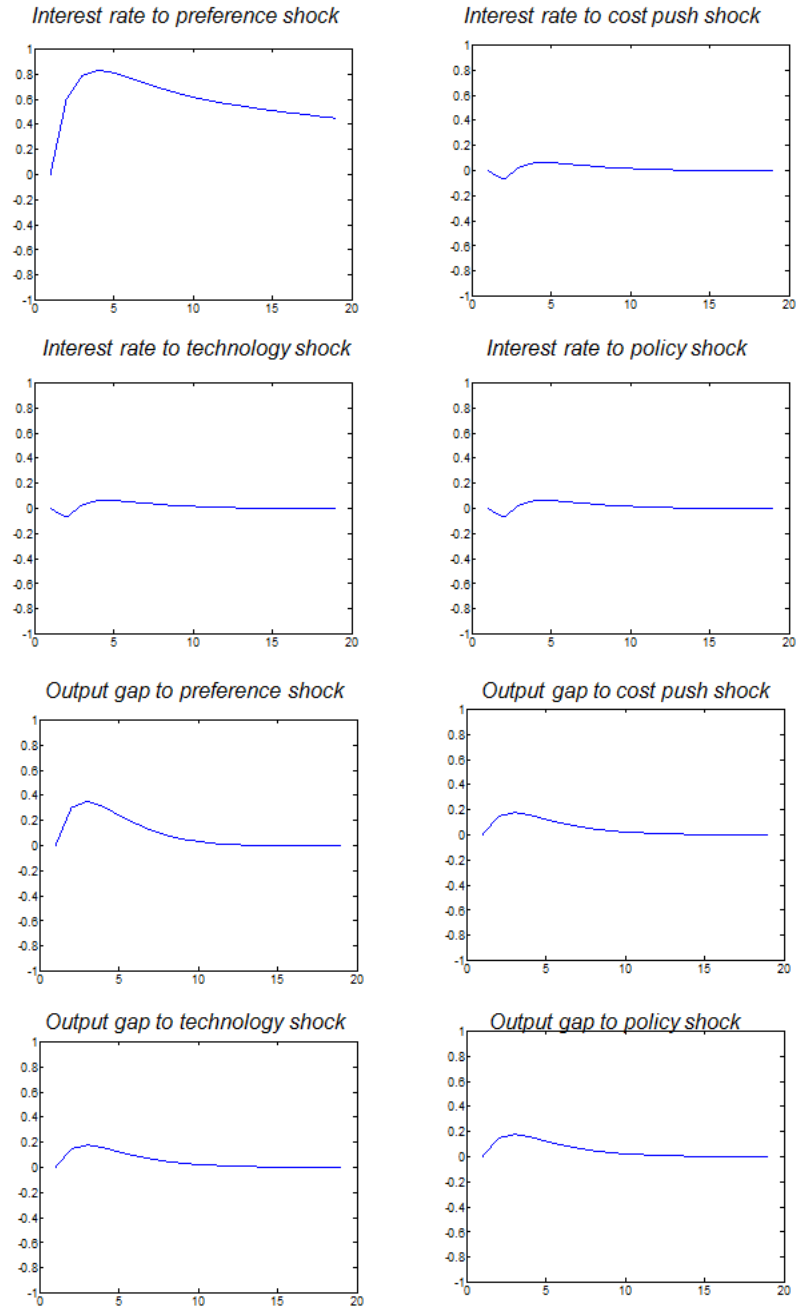


Figure 1.3: Impulse Responses for Scenario 2: Output growth and Inflation

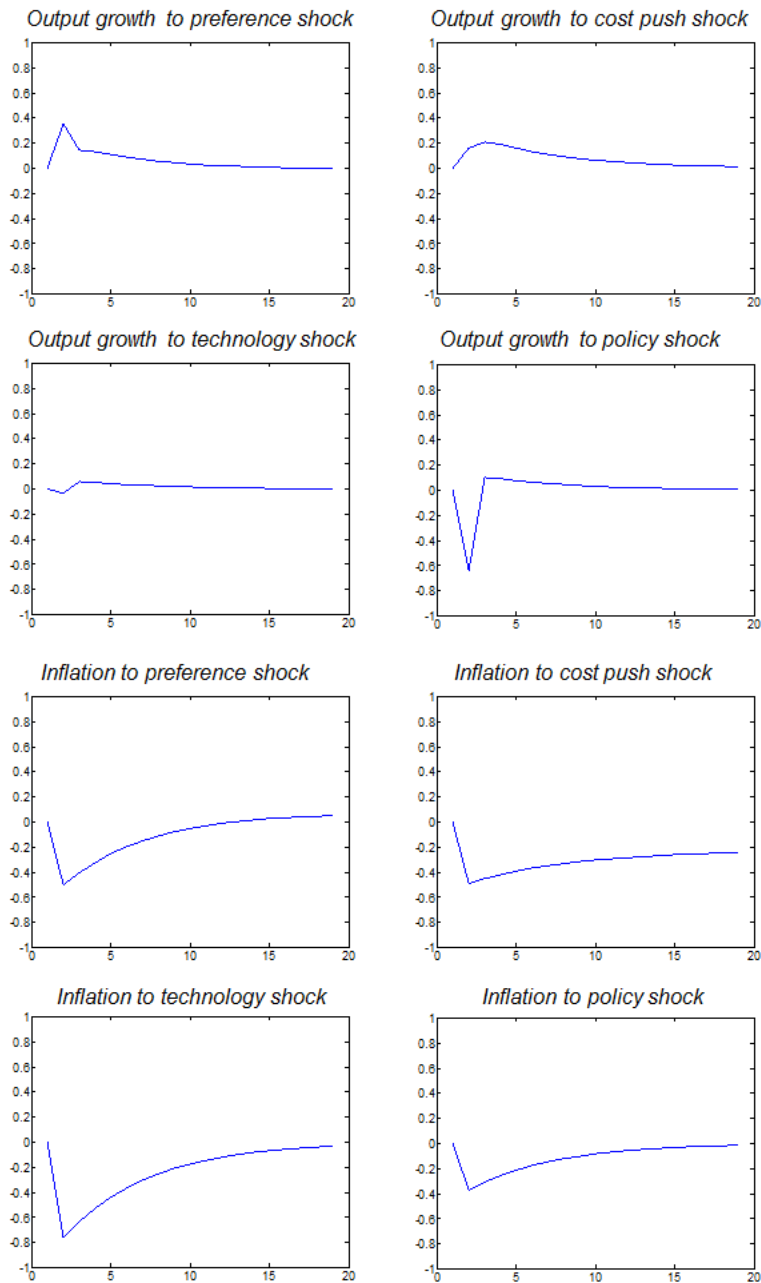
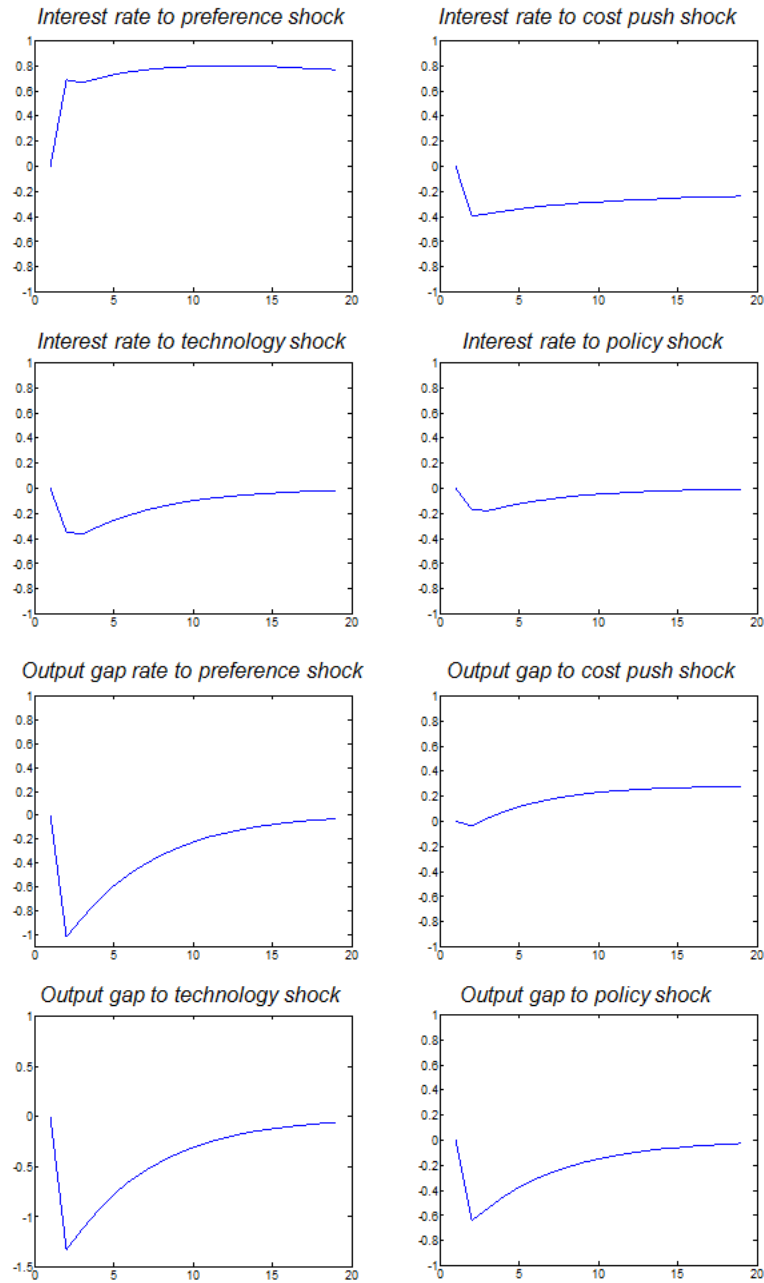


Figure 1.4: Impulse Responses for Scenario 2: Interest Rate and Output Gap



Chapter 2

Analysis of New Keynesian Phillips Curve Relationship in An Estimated Nonlinear DSGE Model

2.1 Introduction

Today's popular New Keynesian Phillips Curve (NKPC) is very different to the original Phillips curve firstly coined by Samuelson and Solow in 1959. This was the time when the policymakers believed that there is a clear trade-off between inflation and output. Stagflations in 1970s and 1980s empirically refute this belief. The rational expectation revolution, led by Robert Lucas and Thomas Sargent, later dominates the macroeconomic theory and practice, which leaves no room for monetary policy.

Keynesian later came up with NKPC with rational expectation and micro-founded macroeconomic theory. Among several versions, each with different version of short run nominal and or real rigidities, pricing a la Calvo (1983) is the most popular one due to its simplicity. It assumes that at any period, there is a fixed probability that a firm will change its price. Policymakers now have a menu tradeoff between inflation and output stabilizations.

Phillips Curve is nonlinear both theoretically and empirically. Its nonlinearity has been agreed upon and many have tried to empirically estimate it. Kuttner & Robinson (2010) tries to understand the empirically flattening Phillips Curve, Tambakis (2009) discusses the optimal monetary policy with a convex Phillips Curve, and Fendel et al. (2011) confirms that professional forecasters empirically include nonlinearity in their Phillips Curve.

Ongoing research on NKPC by both academic and central banks has been based on either the ad-hoc relationship between current inflation on the left hand side and its past and some measurement of economic activity on the right hand side or the reduced form drawn from a DSGE model. Central banks in particular are very interested in the empirical aspects of the curve to be able to use it as one of important tools in conducting their monetary policies in the short to medium term.

The research has been also influenced much by methodological development in the time series econometrics. In the last two decades, unobserved component model, reduced form, two state Markov chain as in Davig (2007) has been one of the leading ideas. In general, it can be assumed that the NKPC can operate in two different states: flat NKPC corresponding to the state where the cost of price adjustment is high, and steep NKPC due to the state of low cost of price adjustment.

There are many methods for solving [nonlinear] DSGE model. Aruoba et al. (2006) provides a comprehensive evaluation of several major solution methods (i.e. undetermined coefficients in levels and in logs, finite elements, Chebyshev polynomials, second and first order perturbations and value function iteration) for dynamic equilibrium economies. They conclude that nonlinear solution methods are generally better than the linear ones.

Rubio-Ramirez & Fernández-Villaverde (2005) and Fernandez-Villaverde & Rubio-Ramirez (2007) are among the first papers that bring the nonlinear DSGE models to the data. Using simulated data, they show how particle filtering facilitates the likelihood-based inference of a nonlinear DSGE model. They show the particle filter results can be used to estimate the structural parameters of the model. The estimation could rely on the classical maximum likelihood or a Bayesian approach. An & Schorfheide (2007) estimates nonlinear smaller scale of New Keynesian DSGE model for the US, also using simulated data. Flury & Shephard (2011) confirms that particle filter,

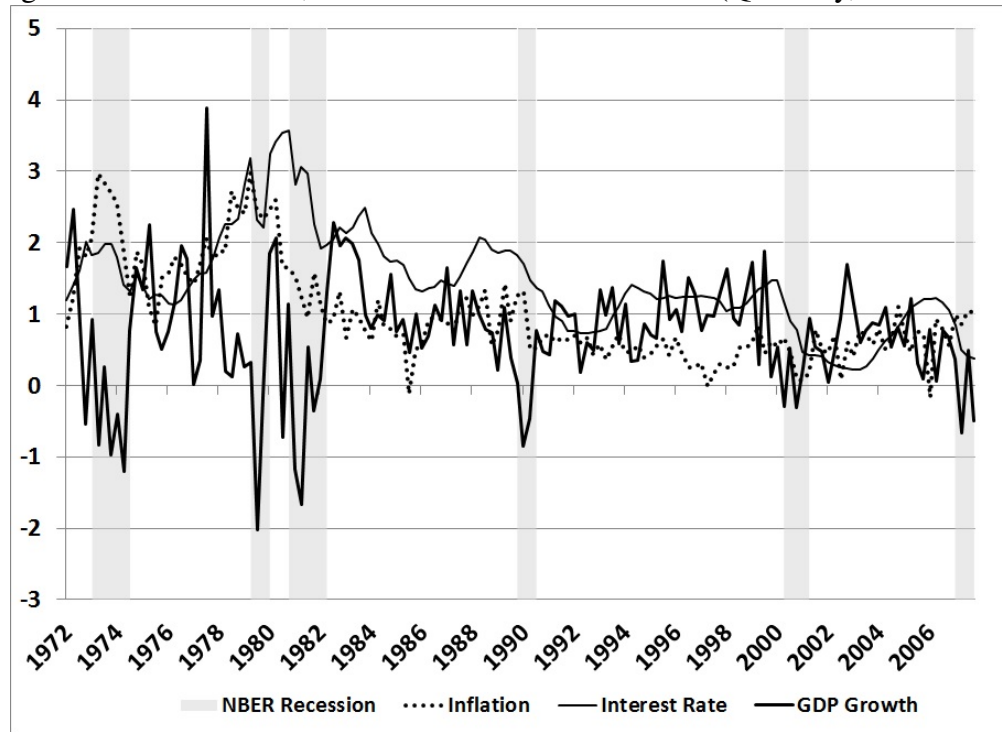
given the model specification, generates unbiased estimates of the likelihood. Bayesian inference based on simulated likelihood can be used to carry out likelihood based inference using its simulations.

This paper presents an empirical proof for nonlinear NKPC relationship for the US economy by allowing enough theoretical foundation including the nonlinearities in a New Keynesian DSGE model presented in Smets & Wouters (2003) and Amisano & Tristani (2010). Practically, instead of log linearizing the nonlinear dynamical system around its steady states, we derive the second order approximation of the system around the same steady states. By taking advantage of recent development in econometrics of nonlinear dynamical system, we use a variant of particle filter within a Bayesian MCMC algorithm to estimate the model parameter values. It is shown that nonlinear model better accommodates the sharp turns from the steady state in the short run. The impulse responses are also shown to be different during low vs. high inflation regimes.

2.2 Some empirical facts

Figure 2.1 shows the data for PCE inflation rate, 3-month Treasury Bill yield, and GDP growth (all quarterly) from 1972:Q4 to 2008:Q3. It covers the periods of stagflation in 1970s and early 1980s up until the last Great Recession of 2007-2008. The highest inflation rate happened in 1980:Q1 at 2.99% (equivalent to 12.5% in annual term), while the lowest happened in 2006:Q4 at -0.17% (equivalent to -0.68% in annual term). The quarterly GDP growth rate has been quite volatile as well, especially in 1970s and 1980s. The highest was in 1978:Q2 at 3.9%, while the lowest was in 1980:Q2 at -2.0%.

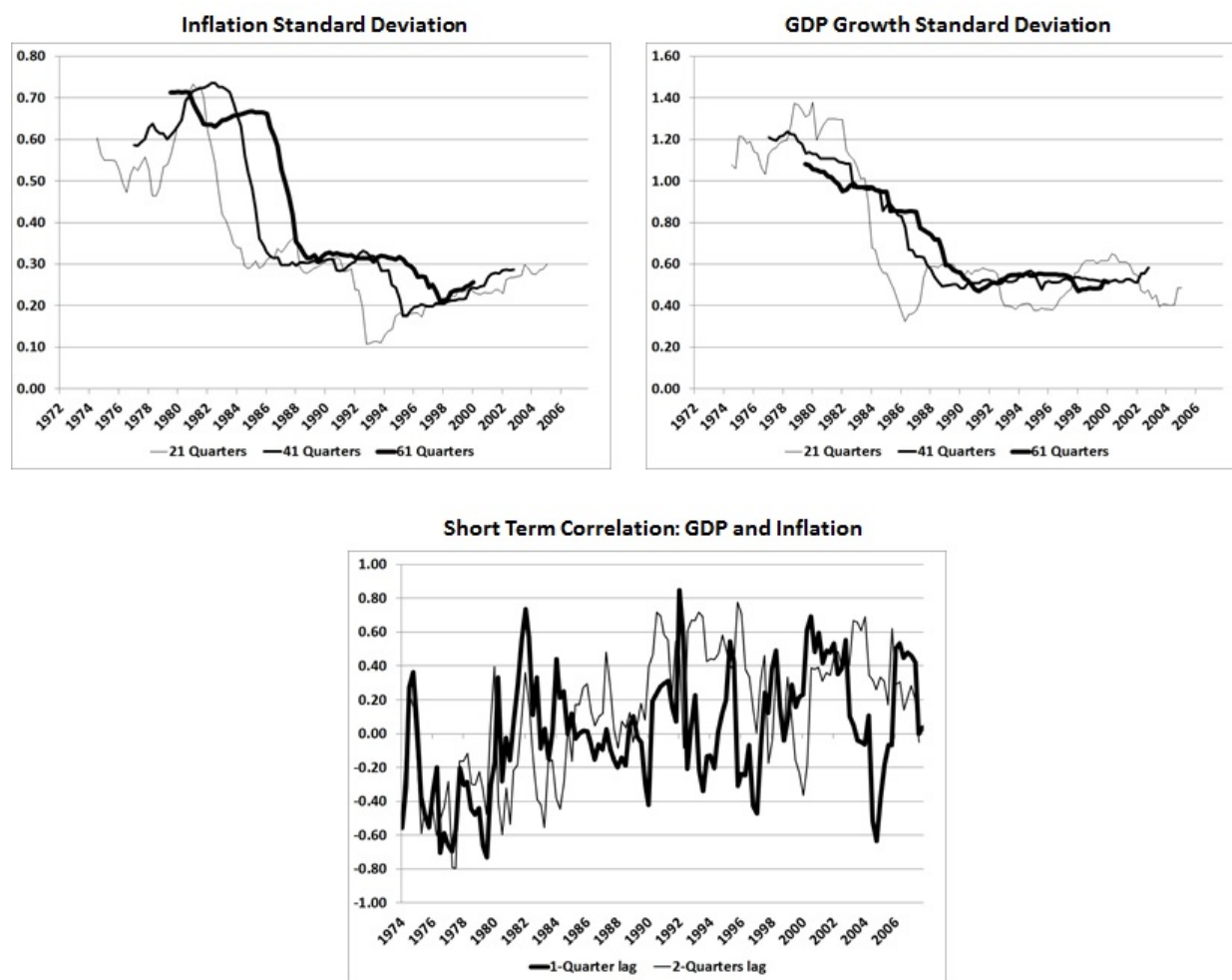
Figure 2.1: US Inflation, Interest Rate and GDP Growth (Quarterly, 1972 - 2008)



Over the scope of the period covered in Figure 2.1 above, both output and inflation are volatile with the output being more volatile among the two. The mean and standard deviation for the inflation are 1% and 0.68% respectively. For output, these numbers are 0.7% and 0.8%. The volatilities are not constant over time. In fact, there is a strong trend of increased stability over the time span covered in the data. The first and the second panels on Figure 2.2 illustrate this.

Traditionally, there is a slight trade-off between inflation and lag of output. See Fuhrer & Moore (1995), for an example. But again, the correlation is not constant over time. The third panel of Figure 2.2 shows the moving correlation between inflation and lagged GDP growth in short term (in this case, 8 quarters). The average of the coefficient of correlations is 0.02 for 1 quarter lag of output and 0.12 for two quarters lag. Interestingly, the coefficients of correlation tend to show a cycle like dynamic.

Figure 2.2: Centered Moving Standard Deviations of GDP Growth and Inflation, and Lagged Short Term Correlation between Inflation and Output



2.3 The Model

The model is based on the standard New Keynesian model as in Woodford (2003), which is extended by Smets & Wouters (2003) and Amisano & Tristani (2010). Woodford (2003) has been such a prototype model that becomes the foundation for larger scale models used to evaluate monetary policy in academic and central banks.

2.3.1 Household

The representative household is maximizing the present value of its lifetime utility which instantaneous utility form is given below.

$$U(C_t, H_t, L_t) = \frac{(C_t - hC_{t-1})^{1-\gamma}}{1-\gamma} - \int_0^1 \chi L_t(i)^\phi di, \quad (2.1)$$

It features a constant relative risk aversion (CRRA), γ . The instantaneous utility depends on current consumption, C_t , habit stock, $H_t = hC_{t-1}$, and aggregate disutility from labor by working in the production of a continuum of intermediate goods producing firms, $\int_0^1 \chi L_t(i)^\phi di$. Consumption C_t is a Dixit-Stiglitz constant elasticity of substitution, θ , aggregate of consumption of the differentiated intermediate goods

$$C_t = \left[\int_0^1 C(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{1-\theta}} \quad (2.2)$$

The constraint for household's maximization problem is its intertemporal budget

$$P_t C_t + B_t \leq \left(1 - \frac{\tau_t}{1 + \tau_t} \right) \int_0^1 \omega_t(i) L_t(i) di + \int_0^1 \Xi(i) di + W_t. \quad (2.3)$$

Each period, the household buys composite consumption goods, C_t and bonds, B_t . At the same time, it receives the aggregate wages, $\int_0^1 \omega(i) L_t(i) di$ adjusted by the tax rate, $\left(1 - \frac{\tau_t}{1 + \tau_t} \right)$. It also receives aggregate profit, $\int_0^1 \Xi(i) di$ while receiving wealth transfer, W_t . Note that, given the Dixit-Stiglitz aggregate of consumption of differentiated goods given above, as a result of consumption cost minimization by the household, price index is given by Dixit-Stiglitz aggregate as follows

$$P_t = \left(\int_0^1 p(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}}. \quad (2.4)$$

Tax rate, $\frac{\tau_t}{1 + \tau_t}$, is written in such a way to guarantee that it falls between 0 and 1. This tax can be distortionary and become a source of inefficiency at the steady state. See Benigno & Woodford (2005) for a comprehensive discussion on distortionary taxes. We assume that τ_t follows an AR(1) process

$$\ln \tau_t = (1 - \rho_\tau) \bar{\tau} + \rho_\tau \ln \tau_{t-1} + v_t^\tau; \quad v_t^\tau \sim N(0, \sigma_\tau^2). \quad (2.5)$$

The first order conditions of household optimization with respect to labor and consumption are given by

$$\begin{aligned} \left(1 - \frac{\tau_t}{1 + \tau_t}\right) \frac{\omega_t(i)}{P_t} &= \frac{\phi \chi L_t(i)^{\phi-1}}{\Lambda_t} \\ \Lambda_t &= (C_t - hC_{t-1})^{-\gamma} - \beta h E_t [(C_t - hC_{t-1})^{-\gamma}] \\ \frac{1}{I_t} &= E_t \left[\beta \frac{P_t}{P_{t+1}} \frac{\Lambda_{t+1}}{\Lambda_t} \right]. \end{aligned} \quad (2.6)$$

2.3.2 Firm

There is a continuum of intermediate goods producers all of whom use the same technology and the industry specific labor is the only factor. The production function of the producer of intermediate good i is given by

$$Y_t(i) = A_t L(i)^\alpha, \quad (2.7)$$

where α is the elasticity of output with respect to labor and where technology, A_t is unit elastic. The technology is assumed to grow exponentially by ρ_a . The log of technology follows an AR(1) stochastic process as shown below.

$$\ln A_t = \rho_a \ln A_{t-1} + v_t^a; \quad v_t^a \sim N(0, \sigma_a^2) \quad (2.8)$$

By assuming Calvo (1983) contract, the firm's optimization problem is to maximize the present value of future profits,

$$\max_{P_t^i} E_t \sum_{s=t}^{\infty} \zeta^{s-t} \beta^s \frac{P_t}{P_{t+s}} \frac{\Lambda_{t+s}}{\Lambda_t} (P_s^i Y_s^i - TC_s^i), \quad (2.9)$$

where ζ is the probability for the firm to change its price in each period. Firms who do not adjust at time t , set their prices as

$$P_t^i = (\bar{\Pi})^{1-\iota} \left(\frac{P_{s-1}}{P_{t-1}} \right)^{\iota}. \quad (2.10)$$

The firm's first order condition is given by

$$\begin{aligned} \left(\frac{P_t^*}{P_t} \right)^{1-\theta \left(1 - \frac{\phi}{\alpha} \right)} &= \frac{\phi \chi \theta}{\alpha (\theta - 1)} \frac{\Omega_{2,t}}{\Omega_{1,t}} \\ \Omega_{2,t} &= \frac{A_t^{-\frac{\phi}{\alpha}}}{\left(1 - \frac{\tau_t}{1 + \tau_t} \right) \Lambda_t} Y_t^{\frac{\phi}{\alpha}} + E_t \zeta \bar{\Pi}^{-\theta \frac{\phi}{\alpha} (1-\iota)} \beta \frac{\Lambda_{t+1}}{\Lambda_t} \Omega_{2,t+1} \Pi_t^{-\iota \theta \frac{\phi}{\alpha}} \Pi_{t+1}^{\theta \frac{\phi}{\alpha}} \\ \Omega_{1,t} &= Y_t + E_t \zeta \bar{\Pi}^{(1-\theta)(1-\iota)} \beta \frac{\Lambda_{t+1}}{\Lambda_t} \Omega_{1,t+1} \Pi_t^{(1-\theta)\iota} \Pi_{t+1}^{\theta-1}. \end{aligned} \quad (2.11)$$

Inflation is defined as $\Pi_t = \frac{P_t}{P_{t-1}}$. On the other hand, the optimal price at time t , P_t^* can be expressed as

$$\frac{P_t^*}{P_t} = \left[\frac{1 - \zeta \left(\frac{\bar{\Pi}^{1-l} \Pi_{t-1}^l}{\Pi_t} \right)^{1-\theta}}{1 - \zeta} \right]^{\frac{1}{1-\theta}}. \quad (2.12)$$

Substituting the last expression into the firm's first order condition will result in

$$\Pi_t = \bar{\Pi}^{1-l} \Pi_{t-1}^l \left[\frac{1}{\zeta} - \frac{1-\zeta}{\zeta} \left(\frac{\phi \chi \theta}{\alpha (\theta-1) \Omega_{1,t}} \frac{\Omega_{2,t}}{\Omega_{1,t}} \right)^{\frac{1-\theta}{1-\theta(1-\frac{\phi}{\alpha})}} \right]^{\frac{1}{\theta-1}}. \quad (2.13)$$

The first order approximation of this last expression is the usual New Keynesian Phillips Curve, where current inflation is a function of expected future inflation. The second order approximation of this equation is too elaborate to be intuitive. See the appendix for a sketch of the second order approximation of this equation. As Amisano & Tristani (2010) points out, Benigno & Woodford (2005) derived the second order approximation of a Phillips curve with a simpler model, where the habit and inflation indexation are not part of the model.

The quadratic approximation of the last equation will be either concave or convex, depending upon the the inflation gap from its steady state. If the inflation is a convex function of the expected inflation, the higher the inflation expected to be from the steady state the higher the faster the firm will increase its price. On the other hand, a negative inflation gap would be responded by a sluggish price adjustment; even more sluggish than the linear version of the Phillips curve itself.

2.3.3 Taylor rule

The Fed's Taylor rule is defined as follows.

$$\begin{aligned}
i_t &= (1 - \rho_i) \left[(\bar{\Pi} - \ln \beta) + \psi_\pi (\Pi_t - \Pi_t^*) + \psi_y (y_t - y_t^n) \right] + \rho_i i_{t-1} + v_t^i \\
\Pi_t^* &= (1 - \rho_\pi) \bar{\Pi} + \rho_\pi \Pi_{t-1}^* + v_t^{\pi^*} \\
v_t^i &\sim N(0, \sigma_i^2) \\
v_t^{\pi^*} &\sim N(0, \sigma_{\pi^*}^2)
\end{aligned} \tag{2.14}$$

The interest rate target, relative to its long run equilibrium, is a positive function of both the inflation gap, $(\Pi_t - \Pi_t^*)$ and the output gap $(y_t - y_t^n)$. The Taylor rule here features interest smoothing by including $\rho_i i_{t-1}$ in the equation. The inflation target is assumed to be stochastic moving average weighted between the steady state inflation and the previous period's inflation target. This assumption, in contrast to a constant inflation target, is more realistic especially considering the clear downward trend of inflation during our sampled period. It means that the Fed is willing to adopt a relatively high inflation target when the actual inflation is high like in 1970s and early 1980s.

2.4 Solving the Model

In general, the equilibrium conditions of the model above - which consists of the Euler equations, the constraints, and the market clearing conditions - can be transformed into a state space model as follows.

$$E_t [f(x_{t+1}, y_{t+1}, x_t, y_t)] = 0, \tag{2.15}$$

where f maps $\mathbb{R}^{2n_x+2n_y}$ into $\mathbb{R}^{n_x+n_y}$. x_t is a vector of state variables (length= n_x). y_t is a vector of non-state variables, including control variables (length= n_y). Note that all variables have been expressed as deviations from their steady state values.

The variables are ordered as follows

$$\mathbf{x} = [y_{t-1} \ i_{t-1} \ \pi_{t-1} \ y_{t-1}^n \ a_t \ \pi_t^* \ \tau_t \ v_t^i]'$$

$$\mathbf{y} = [\Omega_{1,t} \ \Omega_{2,t} \ \pi_t \ i_t \ y_t \ \lambda_t \ \tilde{i}_t \ y_t^n]'$$

Let σ as a scale variable, which will define the second order approximation around the non-stochastic steady state.

Equation (2.15) defines (exact) solution functions g and h whose roles are defined as

$$y_t = g(x_t, \sigma) \tag{2.16}$$

and

$$x_{t+1} = h(x_t, \sigma) + \sigma \varepsilon_{t+1} \tag{2.17}$$

where ε_t is an exogenous, *i.i.d.* sequence of random variable with zero mean and variance matrix Σ .

We use Gomme & Klein (2011) solution method to come up with the second order approximation of the model. The goal is to express g and h as

$$\hat{g}(x) = \sigma^2 k_y + Ax + \frac{1}{2} (I_{n_y} \otimes x') Bx \tag{2.18}$$

and

$$\hat{h}(x) = \sigma^2 k_x + Cx + \frac{1}{2} (I_{n_x} \otimes x') Dx. \tag{2.19}$$

A is the gradient of g with respect to x , while B is the corresponding Hessian. Similarly, C is the gradient of h with respect to x , while D is the corresponding Hessian. On the other hand, k_y is the Hessian of g with respect to σ , and k_x is the Hessian of h with respect to σ .

2.5 State space model and filtering

Measurement equation:

$$y_t^0 = G(x_t, w_t, \theta) \quad (2.20)$$

State equation:

$$x_t = H(x_{t-1}, v_t, \theta) \quad (2.21)$$

where y_t^0 is a subset of imperfectly observable vector y_t^0 ,

θ is the parameter vector,

$v_t \equiv [v_t^a, v_t^{\pi*}, v_t^{\tau}, v_t^i]$ is the structural shocks vector, and

w_t are the measurement errors.

General filtering problem:

given $p(x_t | y_t^0, \theta)$,

obtain $p(x_{t+1} | y_{t+1}^0, \theta); t = 0, 1, \dots, T-1$,

where $y_t^0 = [y_1^{0'} y_2^{0'} \dots y_t^{0'}]'$ contains the observed data up to time t .

This involves 2 steps:

- projection

$$p(x_{t+1} | y_t^0, \theta) = \int p(x_{t+1} | x_t, \theta) p(x_t | y_t^0, \theta) dx_t \quad (2.22)$$

- update

$$p(x_{t+1} | y_{t+1}^0, \theta) = \frac{p(x_{t+1} | y_t^0, \theta) p(y_{t+1}^0 | x_{t+1}, \theta)}{p(y_{t+1}^0 | y_t^0, \theta)}, \quad (2.23)$$

where

$$p(y_{t+1}^0 | y_t^0, \theta) = \int p(x_{t+1} | y_t^0, \theta) p(y_{t+1}^0 | x_{t+1}, \theta) dx_{t+1}. \quad (2.24)$$

Sequential Monte Carlo (SMC) methods are simulation based algorithms needed when working on a models that involve integrals that cannot be solved analytically. SMC belongs to the class of Bayesian methods that use Markov Chain Monte Carlo (MCMC) algorithms. In this method, probability distribution can be approximated from discrete distribution that is built from many weighted random draws (called particles) from the relevant distributions.

Particle filter is basically the general form of Kalman filter and hidden Markov model (HMM). If the state space model is linear and the shocks are normally distributed, the integrations can be performed analytically. In this case, Kalman Filter can be used to facilitate the inference by using maximum likelihood method or Bayesian approach.

In this paper, the state space model is nonlinear. As a result, the integrals cannot be done analytically. Alternatively, we will need to approximate them by using numerical methods. In this study, this paper uses Sequential Monte Carlo methods. The idea is to use a *particle filter* to compute the likelihood $p(y_{t+1}^0 | y_t^0, \theta)$.

Rewriting the updating step above gives us

$$\begin{aligned} p(x_{t+1} | y_{t+1}^0, \theta) &= \frac{p(x_{t+1} | y_t^0, \theta) p(y_{t+1}^0 | x_{t+1}, \theta)}{p(y_{t+1}^0 | y_t^0, \theta)} \\ &= \int p(x_{t+1} | x_t, y_t^0, \theta) p(x_t | y_t^0, \theta) \frac{p(y_{t+1}^0 | x_{t+1}, x_t, y_t^0, \theta)}{p(y_{t+1}^0 | y_t^0, \theta)} dx_t \\ &= \int p(x_{t+1} | x_t, y_t^0, \theta) p(x_t | y_t^0, \theta) dx_t \cdot \frac{p(y_{t+1}^0 | x_{t+1}, \theta)}{p(y_{t+1}^0 | y_t^0, \theta)} \end{aligned}$$

From the updating step,

$$p(x_{t+1} | y_{t+1}^0, \theta) = \frac{p(x_{t+1} | y_t^0, \theta) p(y_{t+1}^0 | x_{t+1}, \theta)}{p(y_{t+1}^0 | y_t^0, \theta)},$$

we can refer to $p(x_{t+1} | y_{t+1}^0, \theta)$ as posterior distribution,

to $p(x_{t+1} | y_t^0, \theta)$ as prior distribution and

to $p(y_{t+1}^0 | x_{t+1}, \theta)$ as the likelihood;

and restate it in a form of Bayes formula

$$p(x_{t+1} | y_{t+1}^0, \theta) \propto p(x_{t+1} | y_t^0, \theta) p(y_{t+1}^0 | x_{t+1}, \theta).$$

2.5.1 Particle Filter

Referring to the last equation,

$$p(x_{t+1} | y_{t+1}^0, \theta) = \int p(x_{t+1} | x_t, y_t^0, \theta) p(x_t | y_t^0, \theta) dx_t \cdot \frac{p(y_{t+1}^0 | x_{t+1}, \theta)}{p(y_{t+1}^0 | y_t^0, \theta)},$$

particle filter can be initiated by taking a sample of size N draws from $p(x_t | y_t^0, \theta)$ distribution,

$$x_t^{(i)} \sim p(x_t | y_t^0, \theta), i = 1, 2, \dots, N.$$

This is called a *swarm of N particles*.

The objective is to get a sample of N draws from $p(x_{t+1} | y_{t+1}^0, \theta)$, i.e. the posterior, by performing 3 steps as follows.

1. (*Projection*) Given $x_t^{(i)}$, simulate the state equation to get

$$x_{t+1}^{(i)} \sim p(x_{t+1} | x_t^{(i)}, \theta), i = 1, 2, \dots, N.$$

Note that this is simulating the prior distribution by drawing N draws from

$p(x_{t+1} | y_t^0, \theta)$ distribution.

2. (*Update*) Assign each draw a weight

$$\omega_{t+1}^{(i)} = p(y_{t+1}^0 | x_{t+1}^{(i)}, \theta), i = 1, 2, \dots, N.$$

Note that this is the likelihood of y_{t+1}^0 conditional on $x_{t+1}^{(i)}$.

3. (*Resample* (with replacement) $x_{t+1}^{(i)}$ using probabilities

$$p_{t+1}^{(i)} = \frac{\omega_{t+1}^{(i)}}{\sum_{j=1}^N \omega_{t+1}^{(j)}}.$$

2.5.2 Conditional Particle Filter

$$\begin{aligned} p(x_{t+1} | y_{t+1}^0, \theta) &= \int p(x_{t+1} | x_t, y_{t+1}^0, \theta) p(x_t | y_{t+1}^0, \theta) dx_t \\ &= \int p(x_{t+1} | x_t, y_{t+1}^0, \theta) p(x_t | y_t^0, \theta) \frac{p(y_{t+1}^0 | x_t, y_t^0, \theta)}{p(y_{t+1}^0 | y_t^0, \theta)} dx_t \\ &= \int p(x_{t+1} | x_t, y_{t+1}^0, \theta) p(x_t | y_t^0, \theta) \frac{p(y_{t+1}^0 | x_t, \theta)}{p(y_{t+1}^0 | y_t^0, \theta)} dx_t \end{aligned}$$

1. (*Projection*) Given $x_t^{(i)}$, simulate the state equation to get

$$x_{t+1}^{(i)} \sim p(x_{t+1} | x_t^{(i)}, \theta), i = 1, 2, \dots, N.$$

Note that this is simulating the prior distribution by drawing N draws from

$p(x_{t+1} | y_t^0, \theta)$ distribution.

2. (Update) Assign each draw a weight

$$\omega_{t+1}^{(i)} = p(y_{t+1}^0 | x_{t+1}^{(i)}, \theta), i = 1, 2, \dots, N.$$

Note that this is the likelihood of y_{t+1}^0 conditional on $x_{t+1}^{(i)}$.

3. Resample (with replacement) $x_{t+1}^{(i)}$ using probabilities

$$p_{t+1}^{(i)} = \frac{\omega_{t+1}^{(i)}}{\sum_{j=1}^N \omega_{t+1}^{(j)}}.$$

2.6 Data, Priors, and Simulation Time

In the estimation, the data that we include are quarterly GDP growth, quarterly PCE inflation rate, and 3-month Treasury Bill yield that cover the period of 120 quarter from 1972:Q3 - 2002:Q3. The 3-month Treasury Bill yield is exponentially transformed into its quarterly rate. The choice of the data coverage is intended to cover the high and volatile inflation rate period of 1970s and early 1980s.

We decide to limit the observation to 120 quarters to keep our simulation time relatively manageable. Fasolo (2012) calculates the optimal number of particles needed in the computation of the likelihood of simulated data from a DSGE model that features nominal and real rigidities. It concludes that the particle size of $N=20,000$ to $N=30,000$ show the largest gain of tracking the state variables in the model, regardless the size of the model and number of state variables.

The next task is determining the length of the Markov Chain in the posterior simulation. Basically, the chain needs to be long enough to achieve convergence in the posterior distribution of the parameters. In this version of paper, we use $M=440,000$ by discarding the first 40,000. Simulation time is a big issue. On a desktop computer with an i7 processor, with $N=10,000$ particles and 19 parameters to be estimated, it takes 1.1 second for the conditional particle filter to calculate the likelihood. With $N=20,000$ particles, it takes 2.1 seconds per iteration.

In the future, the simulation time will not be a significant issue anymore. The development of parallel computing for nonlinear DSGE models like Herbst & Schorfheide (2013) is one of the reasons for this. On the other hand, computer processors are increasingly faster. In addition, these

processors are now featuring more graphics processing units (GPUs). See Aldrich et al. (2011) for the discussion about how they use compute unified device architecture (CUDA) of NVIDIA GPUs for parallelization.

The priors are mostly the same priors used by Smets & Wouters (2003) and Amisano & Tristani (2010). We used a different prior for the tax parameter, $\bar{\tau}$ to match the mean tax rate in the US during the sample period. We also use priors from Del Negro & Schorfheide (2008) for the technology persistence parameter, ρ_a and for the Calvo parameter, ζ for US data. Nevertheless, the results are not different to the original set of parameters used in Amisano & Tristani (2010); it does not change the speed of convergence either. This is in line with the conclusion reached by Herbst & Schorfheide (2013) when they used a more diffused priors based on Smets & Wouters (2007).

2.7 Results

Figure 2.3 and 2.4 show the distributions for the prior and posterior of the estimated parameters in the model by using $N=10,000$ particles and $M=440,000$. Almost all posterior distributions are less dispersed than their respective prior distributions. Several parameters could benefit from longer simulation.

Nevertheless, the posterior means are already stable enough. A simulation with $M=330,000$ produced a very similar RMSE for one step ahead forecast of the data. We also run the simulations with $N=20,000$ particles with $M=330,000$ and $M=550,000$. The results are very similar with the ones reported in this paper.¹ As a comparison, Amisano & Tristani (2010) uses $N=20,000$ and $M=550,000$.

Figure 2.5 shows the actual data used in the simulation and their one period ahead predicted values. The deterministic trend is removed from the output variable. The data for inflation rate and

¹The simulation results with $N=20,000$ and $M=330,000$ or $M=550,000$ are available from the author by request.

Figure 2.3: Posterior and Prior Distributions of Parameters (1)

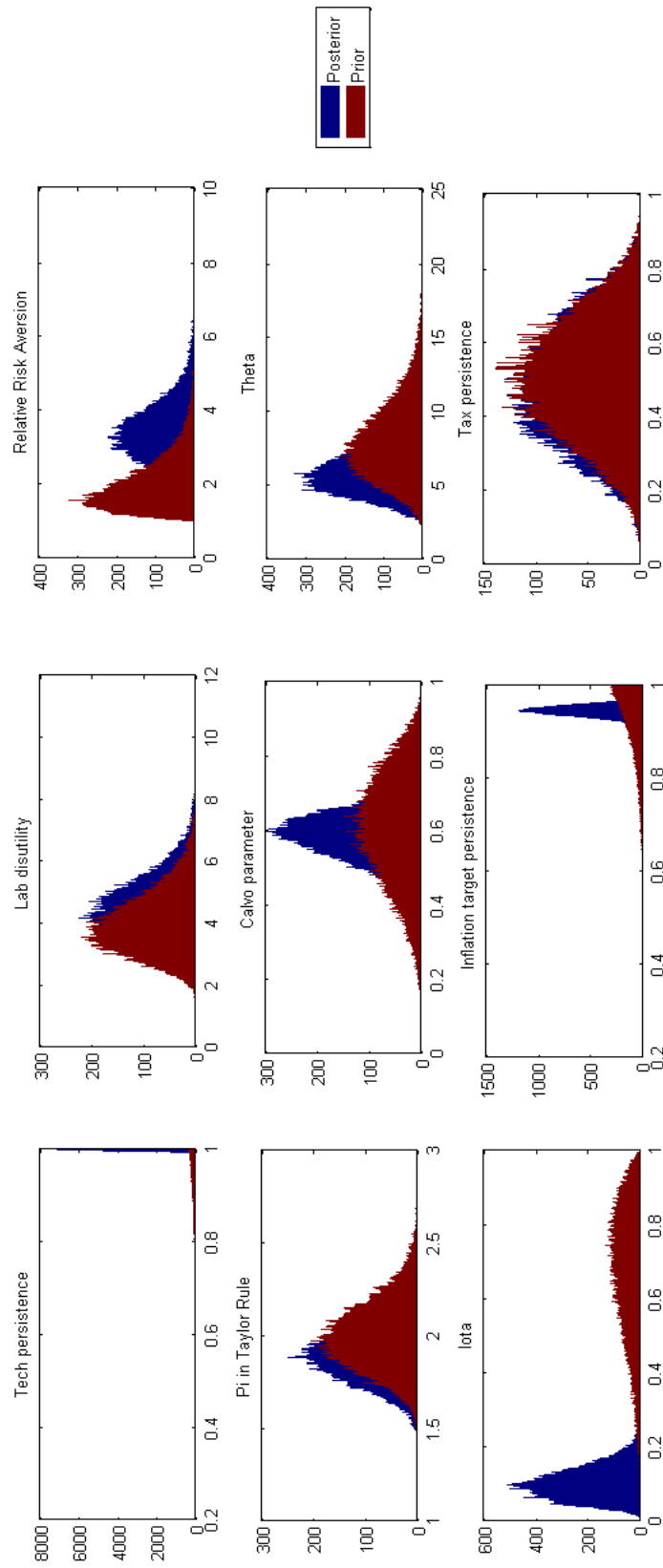


Figure 2.4: Posterior and Prior Distributions of Parameters (2)

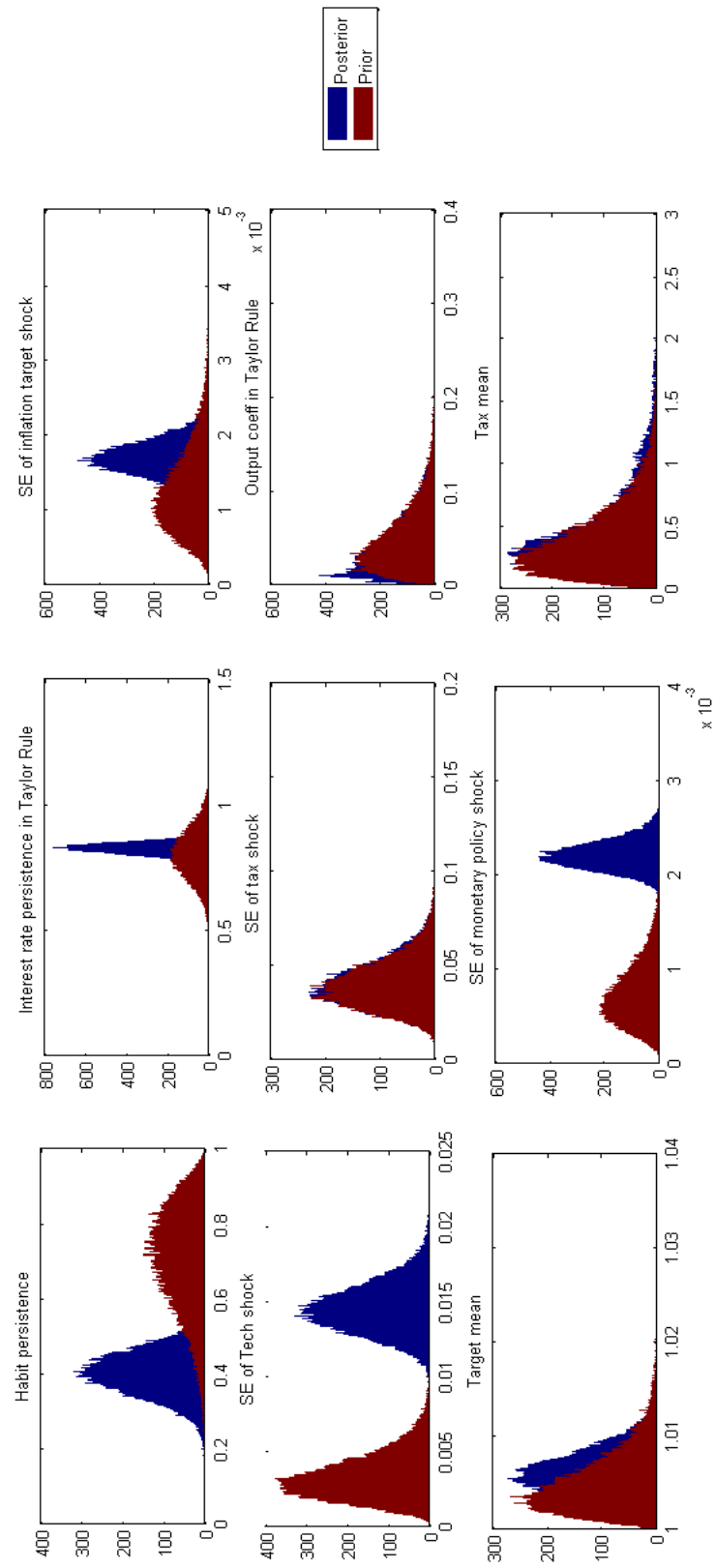
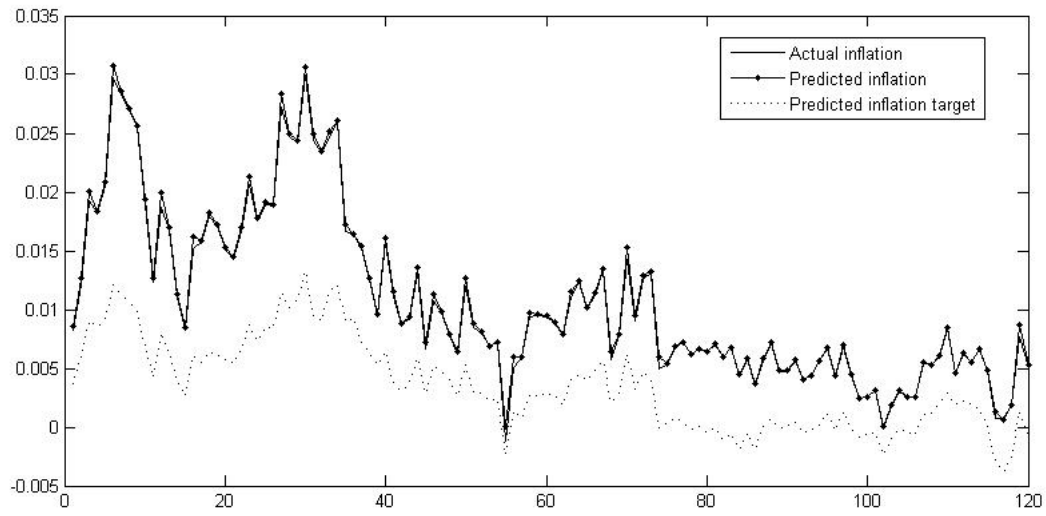


Figure 2.6: Actual inflation, predicted inflation, and predicted inflation target



interest rate are the actual data. It looks like the model fits the data really well.

Figure 2.5: Actual data and fitted value for inflation, interest rate and output

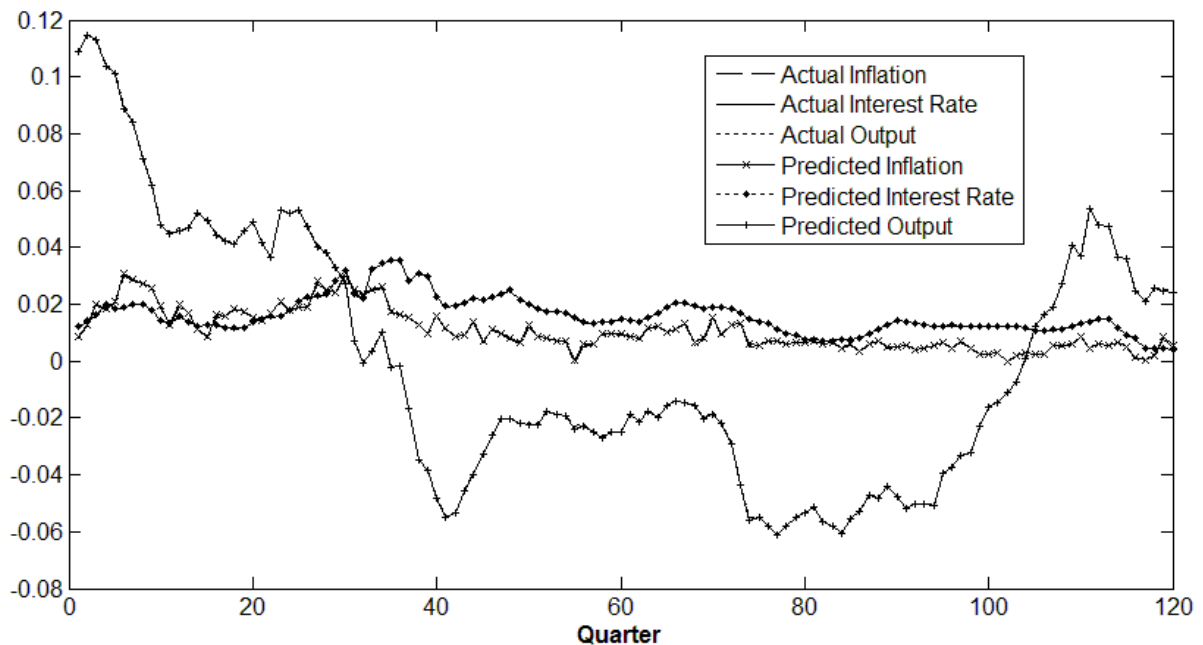


Figure 2.6 shows the actual and predicted values of inflation, along with the estimated inflation target for each period. It clearly shows that the implied inflation target estimated by the model is indeed trended downward; the target is relatively high during high inflation periods.

The results of the posterior simulations are shown in Table 2.1. They are broadly in agreement with the ones suggested by similar model for the US data. Interestingly, the parameter values are quite close to the ones for the Euro area data estimated in Amisano & Tristani (2010). Just like the Euro area parameters, some of the posterior distributions are not narrower than their respective prior distribution. There are 4 out of 19 parameters whose prior coefficient of variations are lower than its posteriors. They are ι (indexation), ρ_π (inflation target persistence), ρ_τ (tax shock persistence), and τ (mean of tax parameter). For the Euro area data, there are 5 out of 19 parameters: γ (relative risk aversion), ϕ (labor disutility), ρ_τ (tax shock persistence), σ_τ (tax shock standard error), and τ (mean of tax parameter).

In terms of parameter values, the intertemporal discount rate, β is slightly smaller than its Euro area counterpart (0.992 vs. 0.994). This indicates a less patient US consumers relative to their European counterparts. The relative risk aversion, γ indicates that US households are less risk averse (3.43 vs. 4.24). The habit persistence, h is lower for the US (0.40 vs. 0.43). Labor disutility coefficient is higher in the US (4.55 vs. 4.36). Calvo parameter, ζ is higher in the US (0.58 vs. 0.45). This means that firms on average change their prices about every 2.4 quarters; less frequently than the Euro area firms who do this about every 1.8 quarters.

The inflation persistence coefficient, ι is surprisingly low even though it is slightly higher than in Euro area (0.10 vs. 0.07). This is in stark contrast to the traditional belief about the inflation being so highly persistent. For example, see Fuhrer & Moore (1995). Amisano & Tristani (2010) argues that it is actually an appealing result given that the indexation is included in the model in an ad-hoc fashion. In his comprehensive survey about the coefficients of New Keynesian Phillips curve, Schorfheide (2008) provides a range of values for this inflation inertia. It ranges from 0 to 4. This is also consistent with the recent development on greater consensus of the decreasing inflation persistence since 1980s due to the great moderation and decreased technology shocks volatility as discussed in Carlstrom et al. (2009). Smets & Wouters (2007) actually finds a comparable estimate for inflation persistence of 0.24 for the US data within a linear model comparable to Smets & Wouters (2003).

Taylor rule parameters are quite similar between the US and Euro area. The coefficient of lagged interest rate, ρ_i is slightly lower in the US (0.82 vs. 0.85). The coefficient for the inflation gap, ρ_π is almost the same between the two economies (1.91 vs. 1.92). This indicates the similar level of distaste toward inflation among the two monetary authorities. On the other hand, the coefficient for the output gap are fairly small for both (0.04 vs. 0.06), indicating a slightly more responsive monetary policy towards the output gap by the European central banks. For both economies, its monetary authorities clearly regard the inflation stability as much more important than output stability, which is consistent with Taylor (1993) principle. The inflation targets are equally persistent for both economies. ρ_i is equal to 0.94 for both.

Table 2.1: Results from Posterior Simulations

Parameters	Posterior				Prior			
	Mean	STD	2.5% Low	2.5% High	Mean	STD	2.5% Low	2.5% High
β	0.992	0.001	0.990	0.995	0.900	0.091	0.664	0.997
γ	3.427	0.762	2.179	5.185	2.000	0.706	1.120	3.771
h	0.402	0.061	0.280	0.518	0.700	0.138	0.400	0.925
ϕ	4.553	0.958	2.976	6.659	3.994	0.996	2.374	6.243
θ	6.017	1.644	3.325	9.715	7.999	2.632	3.815	14.025
ζ	0.580	0.070	0.430	0.705	0.601	0.147	0.299	0.862
ι	0.103	0.044	0.034	0.199	0.666	0.178	0.279	0.947
ψ_π	1.913	0.162	1.621	2.254	2.000	0.184	1.672	2.390
ψ_y	0.044	0.034	0.004	0.130	0.050	0.036	0.006	0.139
ρ_i	0.823	0.024	0.773	0.868	0.800	0.099	0.606	0.994
ρ_τ	0.485	0.156	0.205	0.779	0.500	0.151	0.212	0.788
ρ_a	0.996	0.002	0.990	1.000	0.899	0.092	0.658	0.997
ρ_π	0.943	0.012	0.919	0.966	0.899	0.091	0.660	0.997
$100 * \sigma_\tau$	4.064	1.228	2.017	6.759	3.998	1.268	1.918	6.816
$100 * \sigma_a$	1.451	0.164	1.157	1.796	0.333	0.150	0.106	0.686
$100 * \sigma_\pi$	0.169	0.023	0.128	0.217	0.126	0.056	0.041	0.257
$100 * \sigma_i$	0.221	0.015	0.194	0.253	0.075	0.033	0.024	0.153
$\bar{\tau}$	0.454	0.321	0.066	1.289	0.399	0.280	0.048	1.100
$\bar{\pi}$	1.006	0.002	1.002	1.011	1.005	0.003	1.001	1.013

The impulse responses are depicted in Figure 2.7 and 2.8. Given the fact that the parameter values are relatively close to the ones Amisano & Tristani (2010) estimates for the Euro area data, the impulse responses show that inflation, interest rate, output respond in a similar fashion towards the four different shocks introduced: technology, inflation target, tax, and monetary policy. More importantly, the trajectories are also generally different between the high inflation vs. low inflation starting points. Nevertheless, the magnitudes of the responds are a little different between the two economies.

Gallant et al. (1993) shows that the impulse response functions of a nonlinear dynamic model depend on the initial starting point. These impulse responses are calculated based on the appropriate definition in this context. Suppose that \mathbf{Y} is the expected future paths of the variables conditional on the state \mathbf{x}_t and \mathbf{Y}' is the expected future paths of the variables conditional on the state \mathbf{x}'_t . \mathbf{x}_t and \mathbf{x}'_t are equal to each other except for an individual element that is perturbed by a known shock. Then, the difference between \mathbf{Y}' and \mathbf{Y} is the [nonlinear] impulse response functions. Note that a linear state space model does not need this comparison because the impulse response functions do not depend on the initial state conditions.

We then select two different starting points: 1980:Q1 at 2.99% and 1986:Q2 at -0.12%. Each of these is the highest and the lowest inflation rate respectively in our sample. When doing this we must keep in mind that the nonlinearity in our model is the second order approximation around the steady states. This means that the further we are from the steady states, the less accurate the approximation is. Nevertheless, we can still show two different impulse response functions based on the two scenarios for the starting point.

The strategy is by first calculating the filtered values of the state variables on these dates, namely \mathbf{x}_t and \mathbf{x}'_t . Each of the state vectors is then used to calculate the simulated next period state vectors, i.e. \mathbf{x}_{t+1} and \mathbf{x}'_{t+1} . These vectors are then used to calculate \mathbf{y}_{t+1} and \mathbf{y}'_{t+1} . On the other hand, \mathbf{x}_{t+1} and \mathbf{x}'_{t+1} are then used to calculate \mathbf{x}_{t+2} and \mathbf{x}'_{t+2} , which are used to calculate \mathbf{y}_{t+2} and \mathbf{y}'_{t+2} . We continue this simulation until the desired length of the impulse response function, T , when we have \mathbf{Y} and \mathbf{Y}' , each is a (3×12) matrix. The difference between \mathbf{Y}' and \mathbf{Y} is our impulse

Figure 2.7: Impulse responses: technology and inflation target shocks

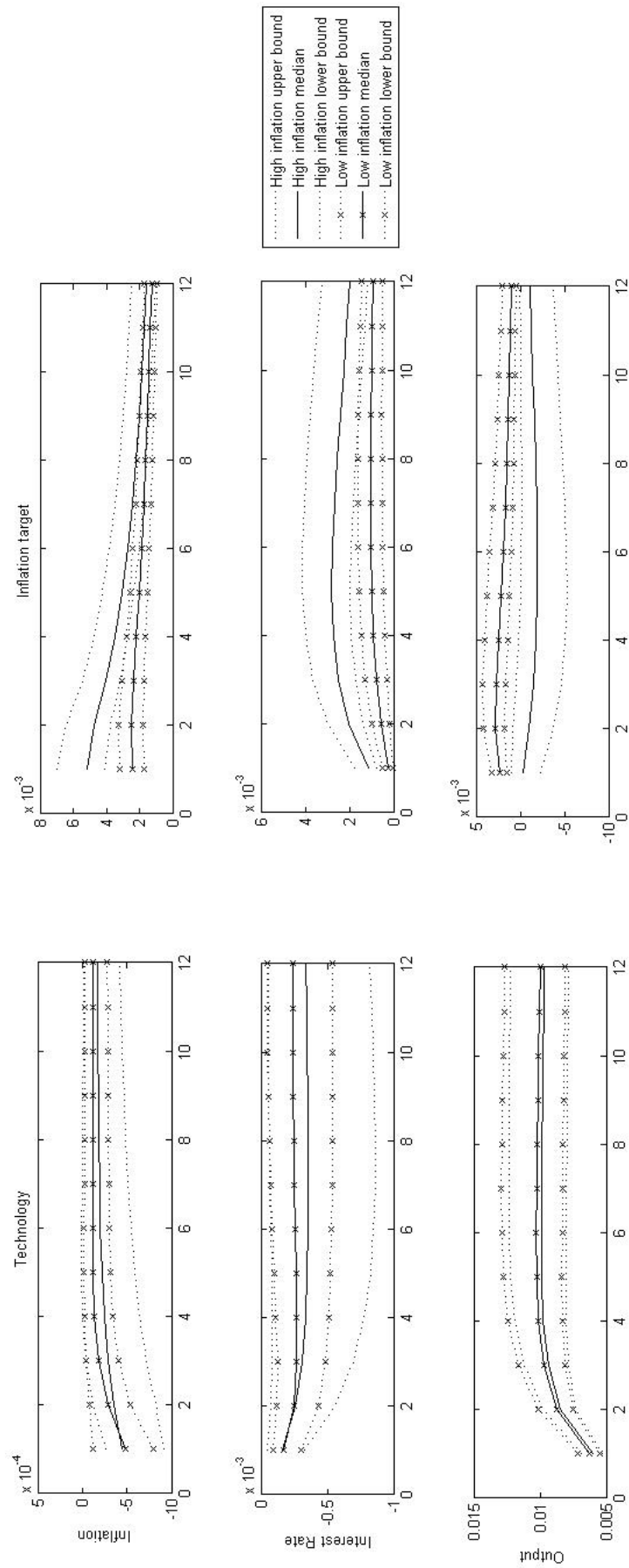
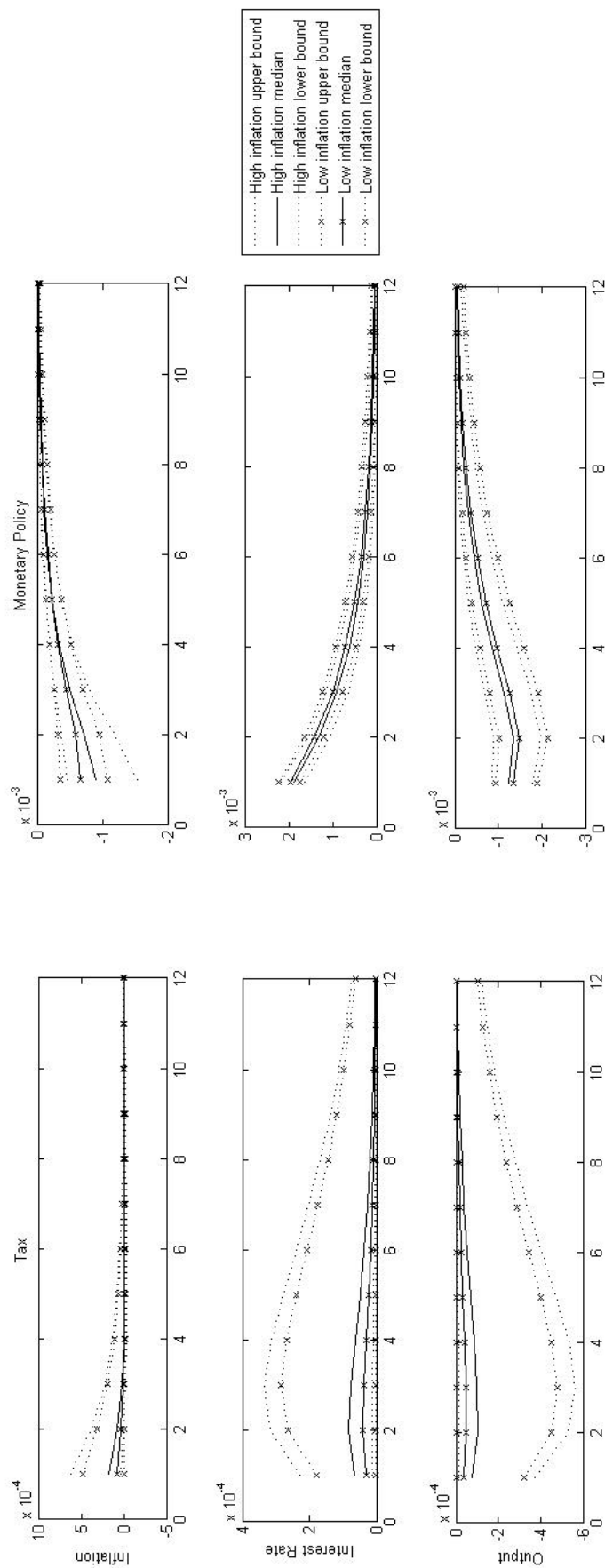


Figure 2.8: Impulse responses: tax and monetary policy shocks



response matrix of 3 by 12 dimension. The rows consist of simulated responses of inflation, interest rate, and output.

Given the simulated posterior distributions for the parameters, we basically have M of vectors of state \mathbf{x}_t at each t . This means that we have the distributions of impulse response functions. Hence, to be more precise, Figure 2.7 and 2.8 show the medians and the 95% confidence intervals of all impulse response functions.

Technology shock has an average of, $\sigma_a = 1.45\%$. As the pictures indicate, any 1 percentage increase in technology immediately brings down the inflation by 0.03 percentage point and interest rate by 0.02 percentage point, while increasing output by about 0.65 percentage point after 4 quarters. The difference between the high inflation and low inflation is not significant.

On the other hand, inflation, interest rate, and output reacts significantly differently towards the inflation target shock between the two starting point scenarios. Inflation target shock has an average of, $\sigma_\pi = 0.17\%$. Figure 2.7 first row second column indicates that for any one percentage decrease in inflation target, if inflation rate is high (low), the inflation will immediately drop by about 3.0 (1.4) percentage points. After 6 quarters, the difference between the two scenarios is no longer statistically significant.

When the inflation is high, as the result of lower inflation target, the interest rate on the other hand will decrease at slower pace than the inflation decreases. This could be attributed to lower risks. As a result the real interest rate will increase. The output's respond could be positive or negative. Looking back at Volcker's anti inflation monetary policy, the economy actually experienced two recessions as a result.

It makes more sense to talk about increasing inflation target when the inflation is low. Note that the low inflation scenario here starts when the inflation rate is -0.12% quarterly (equivalent to -0.48% annualized), which is a quite extreme scenario. If the Fed increases the inflation target by one percentage point, it would increase the interest rate by 0.15 percentage point, which will peak at about 0.62 percentage point in the 7th quarter. Since the inflation will increase faster than the interest rate, the real interest rate will decrease. The impulse response suggests that the output

would increase with hump-shaped trajectory.

It is worth noting that the fact that the indexation parameter, ι is very low means that the inflation dynamic is highly forward-looking. This explains why the inflation rate changes quickly responding to a credible and persistent change in the inflation target.

The tax shock, with average, $\sigma_\tau = 4.06\%$ does not play a big role in the impulse responses. Though statistically significant, any one percentage increase in tax will increase the inflation by less than 0.005 percentage point, increase interest rate by less than 0.002 percentage point, and decrease output almost negligible amount. Moreover, the impulse responses between the two different starting points are not statistically different from each other.

Lastly, monetary policy shock, $\sigma_i = 0.22\%$, in expectation presents two scenarios. Starting from the high inflation, a one percentage increase in the interest rate target will decrease the inflation rate by about 0.4 percentage point. This effect will disappear after about 8 quarters. If the economy starts from the low inflation, the effect will be about 0.3 percentage point. The effect on interest rate is almost the same between the two scenarios. It will increase by about 0.8 percentage point and disappear after about 10 quarters. The effect on output is slightly more pronounced for low inflation starting point. In expectation, output will decrease by 0.6 percentage point when inflation is high. This effect will disappear after about 12 quarters.

2.8 Conclusion

A small scale DSGE model is approximated up to the second order is solved and its likelihood in matching the observed variables for US data is calculated using conditional particle filter. The Bayesian MCMC method is then used to simulate the posterior distributions of the parameters. The estimated values of the parameters are reasonable and fits the data well.

US economy has many similarities with the Euro area economy. In particular, the estimated inflation indexation are both lower than what traditionally believed. As the consequence, the in-

flation is highly forward-looking and adjusts very quickly towards inflation target and monetary policy shocks.

It is confirmed that by a simple nonlinearization, by taking the second order approximation, we can demonstrate the difference of the dynamic changes in the system conditional on the current state of the economy.

Chapter 3

Value at Risk (VaR) Based on GARCH-Type Estimated Volatility Models of 5 Stock Markets

3.1 Introduction

Risk management has been a very fast growing industry since the release of the Basel Accord II in 2004. Basel II The Global Risk Management Survey by Deloitte (2012) shows that the total spending on risk management by financial firms was estimated at \$890 million in 1998. This estimate later has increased to around \$50 billion in 2006 and to around \$100 billion in 2012. As explained in the report, until the end of 2010 the Value at Risk (VaR) is still used by 64% of the financial firms in the management of their fixed income investment.¹

Value at Risk (VaR) is defined as the maximum loss of the market value of an investment over a [short] period of time under a certain confidence level. It is now part of standard practice of any risk management system. Banks, hedge funds, or institutional investors who regularly do trading in the financial market will have VaR component in their periodic risk management analysis and

¹This is down from 73% in 2008. It might be due to some critics to the effectiveness of the method.

reporting.

The tradition started in early 1990s after several high profile bankruptcy cases. Barings PLC who lost \$1.3 billion from derivative trading in 1995. It wiped out all of its \$570 million capital. Orange County lost \$1.8 billion in 1994 due to the interest rate hike. Daiwa lost \$1.6 billion, which is 15% of its capital, from one trader in 1995.

J.P. Morgan, which later became J.P. Morgan Chase, is the first institutional pioneer of the formal use of VaR model in market risk management system. In 1994 in its financial report it announced that its trading VaR is on average \$15 million at 95% level over one day. J.P. Morgan's methodology was increasingly adopted by many banks and financial institutions. J.P. Morgan's RiskMetrics later became a trademark software that every risk manager uses as part of their risk measurement methods.

VaR is the worst loss over a target horizon such that there is a low, prespecified probability that the actual loss will be larger. Define c as the confidence level and L as the loss, measured in positive number. Hence, VaR is the smallest loss, in absolute value, such that $P[L > VaR] \leq 1 - c$. For example, if $c = 0.99$, VaR is then the cutoff loss such that the probability of a greater loss is less than 1%.

To illustrate, suppose we want to calculate the VaR of \$100 million portfolio over 10 days at 99% confidence level. First, mark to market the portfolio (e.g. \$100 million). Measure the volatility of the return, i.e. standard deviation of the return (e.g. 15% p.a.). Set the time horizon or the holding period (e.g. 10 days). Set the confidence level (e.g. 99%). Assuming Normal distribution, this yields 2.33 factor. Assuming that there are 252 trading days in a year, then the VaR over the 10 days is $\$100m \times 15\% \times \sqrt{10/252} \times 2.33 = \$7m$. This means, over the 10 days horizon, there is less than 1% probability for a loss bigger than \$7 million.

This paper estimates the GARCH-based volatility² which is then used for the calculation of several different VaR estimates. In practice, GARCH-based volatility models are not used a lot by

² In statistical term, volatility is defined as the standard deviation of a distribution.

the fund managers. Mehta et al. (2012) reports that historical simulation method and RiskMetrics are among the most popular methods used by banks due to their simplicity.

Quite recently, stochastic volatility models are growing and getting some interests among risk practitioners. Arguably, GARCH models are still among the best volatility models. For example, Lehar et al. (2002) compares the performance of GARCH and stochastic volatility models in pricing and evaluating the volatility of options. They conclude that, despite the fact that stochastic volatility models outperform GARCH in pricing the options, there is no proof that it is also a better model for forecasting the VaR.

In Basel II, banks are allowed to use their own internal model in calculating volatility; hence their own VaR model. The method has to be reported consistently to the supervisory body. In the regulatory context, the VaR model has to be consistent enough that it does not underestimate risks. Underestimating risk means the bank is underestimating the regulatory capital requirement. The regulatory body will then penalize the bank for underreporting their risk. The penalty is in terms of higher requirement for capital in the future. This paper uses backtesting and forecasting error evaluation to analyze the balance between minimizing capital requirement and optimally predicting market risks.

3.2 Literature Review

Markowitz (1952) started the tradition of using the asset returns volatility as a measure of risk. The existing literature has supported that most time series data of financial assets exhibit linear dependence in volatility, which is referred to as volatility clustering in econometrics and empirical finance. Engle (1982) first proposed the ARCH (autoregressive conditional heteroskedasticity) model, which assumes normal errors for asset returns and successfully captures a number of stylized facts of financial assets, such as time-varying volatility and volatility clustering. Later, Bollerslev (1986) proposed a generalized ARCH model by assuming that current volatility depends also

on the past volatility.

The traditional econometric time series models generally assume a normal distribution of stock returns. However, the financial literature has long been aware that financial returns are non-normal and tend to have leptokurtic and fat-tailed distribution. For example, see Mandelbrot (1963) and Fama (1965). In this context, several distributions for returns innovation have been proposed to take into account the excess kurtosis. Bollerslev (1987) applied the GARCH-t model, which assumes that the residual of asset returns follows the Student's t-distribution in order to capture fat-tailed characteristic of time series data. Non-Gaussian time series thus begun to receive considerable attention and forecasting methods have been developed gradually.

3.3 Stylized Facts of Stock Market Daily Log Return Distributions

We use daily data from five stock market indices that covers 8/7/2000 to 6/22/2012 (3071 days). Due to different number of holidays, the actual number of observations in our sample are not the same across the five markets.

1. The S&P/ASX 300 index, henceforth **AXKO**, is a market-capitalization weighted and float-adjusted stock market index of Australian stocks listed on the Australian Securities Exchange. The AXKO index itself is managed by Standard & Poor's. It includes stocks of 200 biggest companies in terms of market capitalization, the S&P/ASX 200 index, and an additional 100 smaller companies, making a total of about 300 components in the index.
2. The Hang Seng Index, henceforth **HSI**, is a freefloat-adjusted market capitalization-weighted stock market index in Hong Kong. It covers stocks of 48 biggest companies listed in Hong Kong stock market. These 48 stocks capture around 60% of the Hong Kong Stock Exchange

market capitalization.

3. The FTSE 100 Index, also called FTSE 100, **FTSE**, or, informally, the "footsie", is an index of the stocks of 100 biggest companies in terms of market capitalization listed on the London Stock Exchange. It is maintained by FTSE, a British provider of stock indices and financial market data. FTSE 100 is one of the most highly cited index in the world.
4. BM&FBOVESPA is a Brazilian company, created in 2008, as a merger between the São Paulo Stock Exchange (Bolsa de Valores de São Paulo) and the Brazilian Mercantile & Futures Exchange (Bolsa de Mercadorias e Futuros). **BVSP** index covers about 80% of the market capitalization.
5. S&P 500 is the mostly used benchmark of the U.S. stock market. **GSPC** is one of its ticker symbols. It is a market-capitalization index that comprises of 500 stocks traded in the New York Stock Exchange.

These five indices are traded in five different stock markets which are located in different time zones: AXKO (GMT+10), HSI (GMT+8), FTSE (GMT+0), BVSP (GMT-3), and GSPC (GMT-6). The characteristics of the markets are not too different from each other. These stock markets are among the top 10 stock exchanges with the highest market capitalization in the world.

Figure 3.1 shows the daily log returns of these indices. It is obvious that these markets are somewhat correlated with each other. Particularly, in terms of their volatility, they all show similar pattern of increasing or decreasing in volatility around the same time periods. All of these markets indicate a slightly decreasing volatility from 2000 to around 2006. In the wake of the global financial crisis in 2008, all markets show a sudden heighten volatility.

The daily log return of each of the five indices has a distribution that features statistically zero mean, slight negative skewness (except for HSI), and high excess kurtosis. Table 3.1 provides the summary statistics of these five markets. These distributions are clearly not Normal.

Figure 3.1: Daily Log Return of 5 Stock Markets, Aug-2000 to Jun-2012

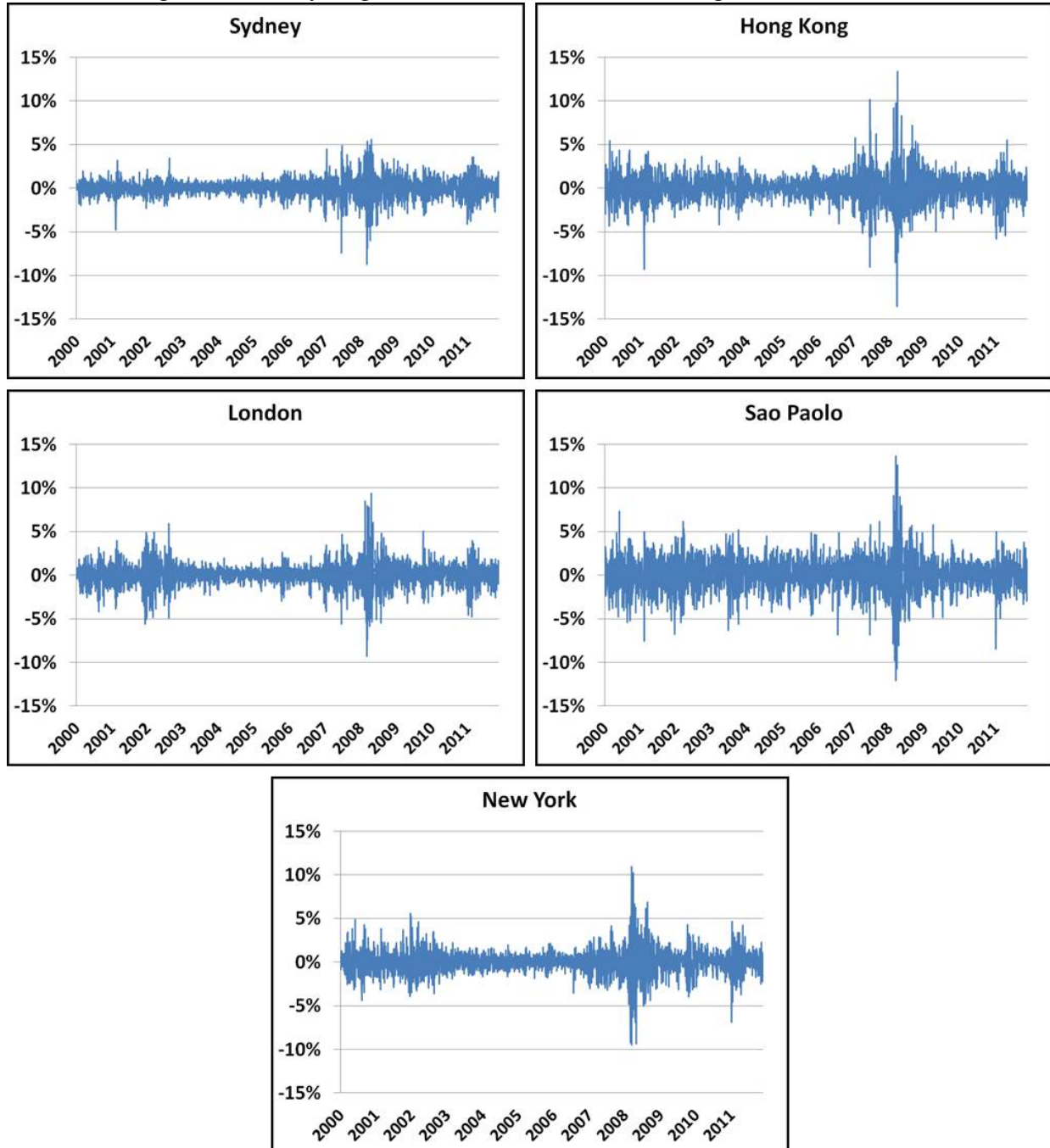


Table 3.1: Summary Statistics of The Distribution of Daily Log Returns

	AXKO	HSI	FSTE	BVSP	GSPC
Mean	0.00%	0.00%	-0.01%	0.04%	0.00%
Standard Deviation	1.06%	1.61%	1.31%	1.90%	1.36%
Skewness	-0.49	0.02	-0.12	-0.11	-0.11
Excess Kurtosis	6.10	8.21	6.02	4.09	7.75
# Trading days	2918	2876	2933	2815	2909

Figure 3.2: Quantiles of Daily Log Returns Against Quantiles of Their Respective Associated Normal Distribution

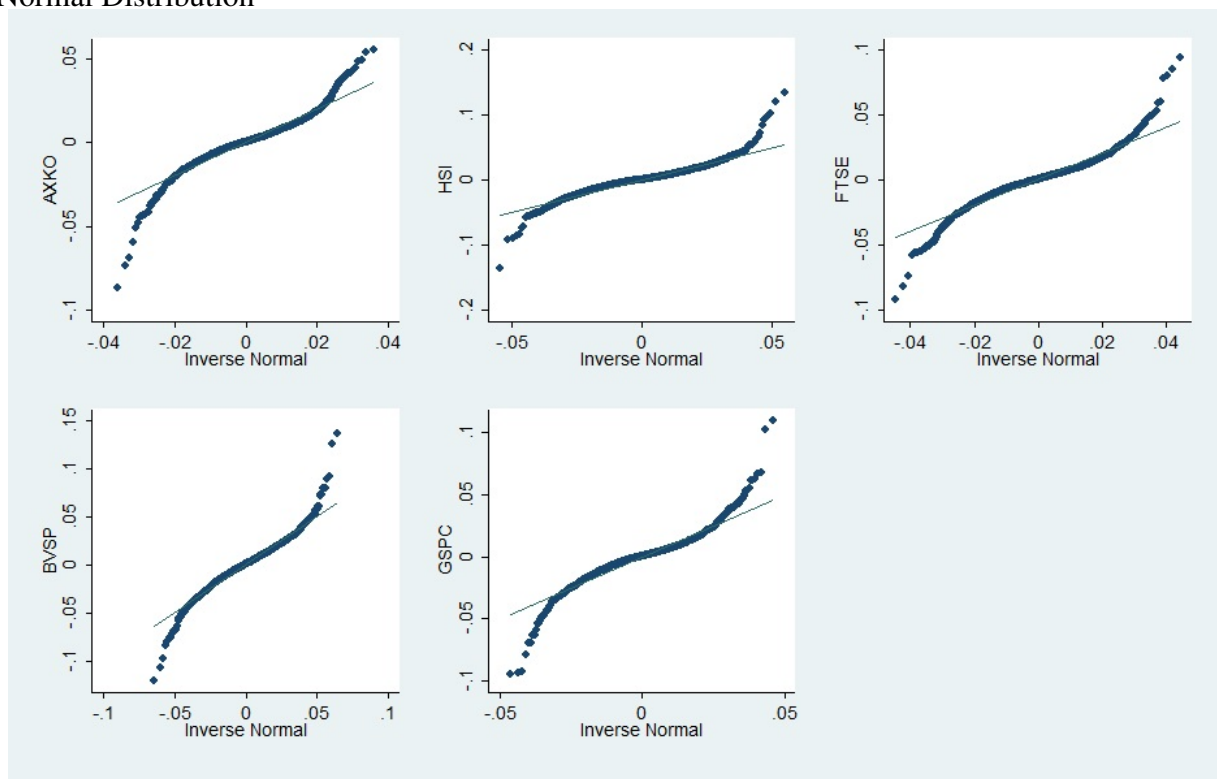


Table 3.2: Normality Tests

Variable	Obs	Shapiro-Francia Test				Skewness-Kurtosis Test		
		W'	V'	z	Prob>z	Pr(Skewness)	Pr(Kurtosis)	Prob>chi2
AXKO	2918	0.932	80.886	6.203	0.000	0.000	0.000	0.000
HSI	2876	0.922	92.755	6.348	0.000	0.707	0.000	0.000
FTSE	2933	0.930	82.706	6.199	0.000	0.010	0.000	0.000
BVSP	2815	0.962	44.190	5.864	0.000	0.019	0.000	0.000
GSPC	2909	0.913	102.592	6.369	0.000	0.012	0.000	0.000

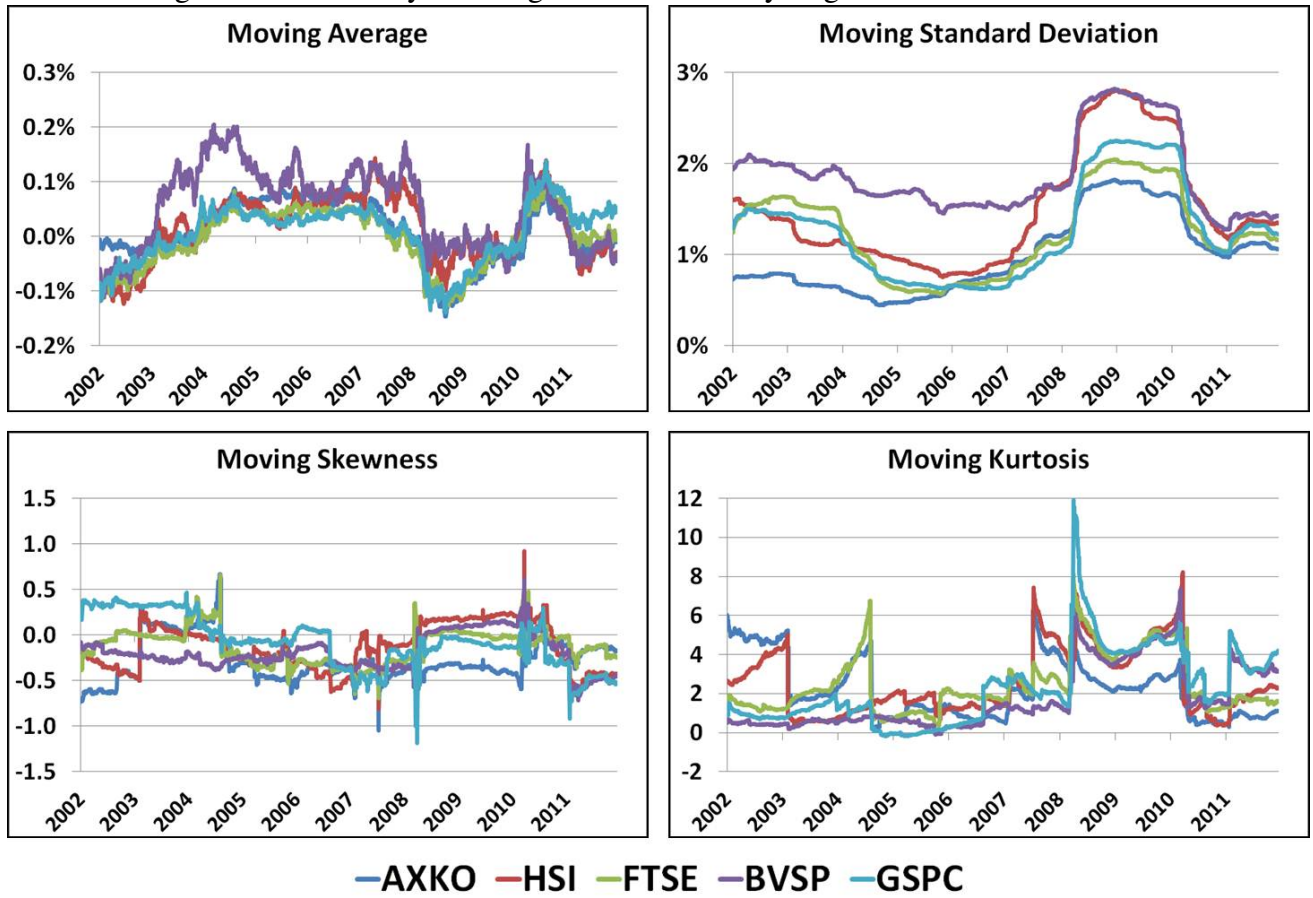
Figure 3.2 shows the plots between the daily log returns of each index with their respective associated Normal distribution. Each plot shows how each of the empirical distributions has a lot of outliers. Table 3.2 provides the numerical tests for the Gaussianity of the empirical distributions. The Null hypothesis is rejected for all of them.

Figure 3.3 shows the dynamics of moving medium term moving average, volatility, skewness, and excess kurtosis of daily log return distributions of the five different markets. The statistics for each date are calculated using the data of the past 1500 trading days, which is equivalent to 6 years period. The moving standard deviation again indicating a stabilizing trend from 2000 to around 2006. The markets' tend to show a small but persistent negative skewness. The non-negativity of skewness coefficient for Hong Kong market is caused mainly by such a very positively skewed distribution of log returns in 2010. Lastly, the excess kurtosis for all of the markets are consistently high. Notably, during periods of economic crises, the distributions became very fat-tailed.

3.4 Methodology

In addition to the Non-Gaussianity of the distributions, they also exhibit the so-called *clustered volatility*. Figure 3.1 shows the daily movement of the five stock market indices. The returns that swing relatively further from the zero means tend to cluster together. For example, we can see clearly that the period of 2003-2006 can be generally considered as tranquil period for all markets. The market volatility in 2008 on the other hand is clearly much higher due to the global financial

Figure 3.3: 1500 Days Moving Statistics of Daily Log Return of 5 Markets



crisis.

We will use five variants of GARCH models: GARCH(1,1), EMWA-RiskMetrics, TARCH(1,1), PARCH(1,1), and EGARCH(1,1). Except for the EMWA-RiskMetrics method, we will also allow the error terms distribution to follow Student t-distribution in addition the Normal distribution. This means that we will have 9 GARCH models to estimate.

Engle (1982) introduced the Autoregressive Conditional Heteroskedasticity (ARCH) model to fit the dynamics of inflation in the United Kingdom. Since then, the method has overwhelmingly dominated the literature on modeling any time series that has ARCH structure.

Due to the nonlinearity of the model and the number of parameters to be estimated, the estimation could be burdensome. We decide to use the simple (1,1) version for each variant. GARCH(1,1) model is given as

$$\begin{aligned} r_t &= \mu + e_t \\ e_t &= v_t \sigma_t, & v_t &\sim N(0, 1) \\ \sigma_t^2 &= \alpha_1 e_{t-1}^2 + \beta \sigma_{t-1}^2 \end{aligned} \tag{3.1}$$

where σ_t^2 denotes the conditional variance since it is a one-period ahead estimate for the variance. RiskMetrics, introduced by Longstaffe & Spencer (1996) has the same form as GARCH(1,1) model. But, it restricts that $\alpha_1 + \beta = 1$. This means that the variance equation will take the form of $\sigma_t^2 = (1 - \lambda)e_t^2 + \lambda \sigma_{t-1}^2$, where λ is called decay factor. It tells us how much the current volatility affected previous period's volatility. The closer the decay factor to 1, the less it is affected by the most recent return. This is the reason why RiskMetrics is called exponentially weighted moving average (EWMA) model. In our study, we will estimate this model as a restricted GARCH(1,1) model with the aforementioned restriction.

TARCH(1,1) is based on the proposed asymmetric GARCH model by Glosten et al. (1993). The volatility model takes the form of

$$\sigma_t^2 = (\alpha_1 + \gamma \mathbf{1}_{\{e_{t-1} < 0\}}) e_{t-1}^2 + \beta \sigma_{t-1}^2, \tag{3.2}$$

where $\mathbf{1}$ is an indicator equaling one when the previous period's return is below some threshold, in this case 0. As the result, this volatility model allows the asymmetric treatment between bad news ($e_{t-1} < 0$) and good news ($e_{t-1} \geq 0$). The bad news is assumed to worsen the volatility more than the good news. The inclination of equity volatilities to rise more when past returns are negative leads to $\gamma > 0$.

EGARCH(1,1) model is based on the exponential GARCH model, by Nelson (1991), which presents the log of variance as follows

$$\ln(\sigma_t^2) = \alpha_1 (|\varepsilon_t| - E[|\varepsilon_t|]) + \gamma \varepsilon_t + \beta \ln(\sigma_{t-1}^2), \quad (3.3)$$

where $\varepsilon_t = e_t/\sigma_t$. The leverage effect is manifested in EGARCH as $\gamma < 0$. Similar to TARCH model, a bad news destabilizes the volatility. A good news on the other hand stabilizes it.

Asymmetric power ARCH (PARCH), based on Higgins & Bera (1992), suggests that the volatility evolves according to

$$\sigma_t^\delta = \alpha_1 (|e_t| - \gamma e_t)^\delta + \beta \sigma_{t-1}^\delta. \quad (3.4)$$

Ding et al. (1993) shows that serial correlation of absolute returns is stronger than the squared terms. As a result, the free parameter δ can capture volatility dynamics flexibly, while asymmetry is captured by γ .

3.5 Estimation

We estimate the nine models above for each sample from our five different markets. We do the estimation with the model evaluation (discussed in the next section) in mind. For each model in each market, we use two different time periods for the sample. The first sample starts from 8/7/2000 and ends on 12/31/2007. This covers a total 1,859 trading days including some missing data. The missing data are due to the difference holidays across the five markets.

The second sample starts on 8/7/2000 and ends on 6/23/2011. This covers a total 2739 trading

days including some missing data. When estimating the models using the second sample, there are some issues with convergence. The parameters for some of the PARCH(1,1) and GARCH(1,1) models do not converge. To take care of this issue we decide to introduce a dummy variable for all 2008 trading days. To maintain comparability between models, we introduce the dummy variable in all models for all markets. The coefficients for the 2008 dummy variable turn out to be negligible in magnitude in all estimation results. But, they are statistically significant for the models where the dummy variable was needed for taking care of convergence issues. The estimation results for the second sample are displayed in Table 3.14 through 3.23.

3.6 Risk model evaluation

We will evaluate the models by conducting out-of-sample forecasts for each model from the estimation results above. The forecasts are for the one-day-ahead volatilities for 250 trading days. This is the typical number of trading days in a year. We do this forecast for each market for two different market situation scenarios: (1) a stressed market and (2) a normal market. The stressed market will be represented by 250 days in 2008. On the other hand, a 250 days period that starts in mid-2011 and ends in mid-2012 will represent a normal market situation.

3.6.1 Loss function

Before evaluating the performance of the volatility models in terms of their ability to forecast the VaR, we want evaluate their performance in terms of their forecasting ability. The loss function can be approached simply by calculating the square root of the mean-square-error (MSE) of the volatility forecasts.

$$MSE : \quad L(\hat{\sigma}_t^2, h_{t|t-k}) = (\hat{\sigma}_t^2 - h_{t|t-k})^2 \quad (3.5)$$

We calculate the RMSE for each of the nine models for each market for two scenarios: (1) stressed market in 2008 and (2) normal market of mid-2011 to mid-2012. A meta-analysis is then used by ranking the models within each scenario for each market. We take the sum of the ranks

each model gets for each scenario. The second table in Table 3.3 summarizes the ranking of the models based on the sum of their ranks in the RMSE evaluations.

3.6.2 Backtest

The out-of-sample forecasted volatilities are then used to calculate the forecasted daily VaRs based on the assumed form of the error distribution for each of the models³. If the volatility model is perfectly calibrated, most of the daily returns would be within the confidence interval of its prediction based on its predicted volatility with an acceptable number *exceptions*, i.e. the observations that fall outside the confidence intervals. For example, with 95% confidence interval, on average, there must be 5% of the observations fall outside the confidence intervals. The more frequently *exceptions* occur the more the model underestimates the risk. As a result, the risk-based capital allocation would be underestimated as well. In addition to being overly exposed to risk, the regulator would impose penalties for too many *exceptions*.

There are several methods for backtest of the risk models: unconditional, conditional. In the light of the recent great recession, models that focus more on the tail risks gain a lot of attention. A simple practical approach like the expected tail-losses is also very useful. See for example Wong (2010).

Among these methods, Kupiec (1995) is the most widely used one. It is popular for a very good reason. The Basel II backtest procedure for VaR is clearly based on this method. This method sees the backtest problem as a series of successes vs. failures. If forecasting horizon is T and the number of failures, i.e. the number of the occurrence of the *exceptions*, is N , then N/T is the failure rate. Ideally, with $(1 - \rho)$ confidence intervals, $N/T = \rho$. A failure rate higher than ρ could indicate an underestimated risk.

An observed failure rate higher than ρ does not automatically qualify the risk is being statisti-

³ In principle, albeit informally, we are simulating the stressed-VaR evaluation for the 2008 great recession scenario.

Table 3.3: Model Rankings

Sum of ranks (Based on # of exceptions)				Sum of ranks (based on RMSE)			
2008		2012		2008		2012	
TARCH(1,1)-t	7	EGARCH(1,1)-t	6	EGARCH(1,1)	8	EGARCH(1,1)	6
GARCH(1,1)-t	11	GARCH(1,1)-t	8	EGARCH(1,1)-t	10	EGARCH(1,1)-t	12
PARCH(1,1)-t	11	PARCH(1,1)-t	9	TARCH(1,1)	16	TARCH(1,1)	17
EGARCH(1,1)-t	12	TARCH(1,1)-t	9	TARCH(1,1)-t	19	PARCH(1,1)	21
RiskMetrics	16	RiskMetrics	17	PARCH(1,1)	25	GARCH(1,1)	22
TARCH(1,1)	16	EGARCH(1,1)	18	GARCH(1,1)	30	TARCH(1,1)-t	22
EGARCH(1,1)	23	GARCH(1,1)	18	PARCH(1,1)-t	31	GARCH(1,1)-t	26
PARCH(1,1)	23	PARCH(1,1)	18	GARCH(1,1)-t	34	PARCH(1,1)-t	26
GARCH(1,1)	24	TARCH(1,1)	18	RiskMetrics	43	RiskMetrics	37

cally underestimated by the risk model. Some of the failures could simply be caused by bad luck. To statistically test the failure rate, Kupiec (1995) develops approximate 95% confidence regions for the test.⁴ The regions are defined by the tails points of the log-likelihood ratio as follows.

$$LR = -2\ln \left[(1 - \rho)^{T-N} \rho^N \right] + 2\ln \left[(1 - (N/T))^{T-N} (N/T)^N \right] \quad (3.6)$$

When T is large, LR is distributed chi-square with one degree of freedom under the null that ρ is the true probability. This means that we would reject the null if $LR > 3.841$.

In Table 3.4 through 3.8 we present the results of backtest for the period of mid-2011 to mid-2012. On the other hand, Table 3.9 through 3.13 present the backtest results for 250 trading days in 2008.

Forecast Error

- In terms of the forecast error, models that accommodate asymmetry are generally the better than the ones that assume symmetry. In particular, the exponential GARCH models are generally the best among the 9 models we use in this paper. Brazilian market is an *exception*. For BVSP, the threshold ARCH model is the best for 2008 while GARCH is the best for mid-2011 to mid-2012. Due to its simplicity, RiskMetrics consistently misprices the market indices compared to all other models evaluated here.

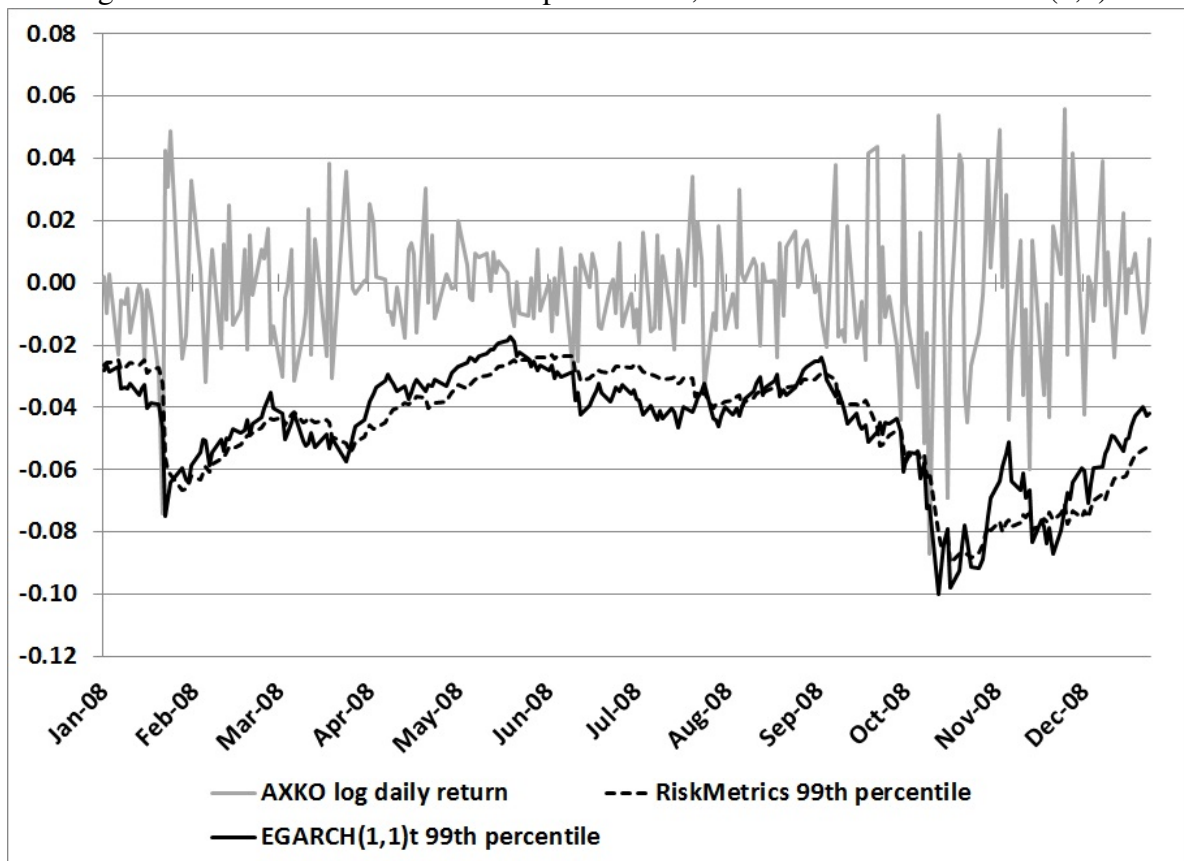
⁴For a detailed discussion about this backtest method, please refer to Jorion (2007).

- For each of our estimated models, the one with assumed t-distribution for the error terms is not more precise compared to its Normal distribution counterpart. This is to be expected. The fact that both Normal distribution and t-distribution are symmetric, the central tendency of the distribution between the two distributions are theoretically the same. The story will be different when we compare the extreme values.

Number of exceptions

- Despite the weakness of the test, there are some clear conclusions we can take. As predicted, the t-distribution is better than Normal distribution in tracking the extreme values of the daily log returns. All of the t-distribution models are among the best in terms of number of *exceptions*. This is true both in normal time and stressed time.

Figure 3.4: AXKO: 2008 Out of sample forecast, RiskMetrics vs. EGARCH(1,1)-t



- During the normal time, EGARCH(1,1)-t does the best job in predicting the extreme values. PARCH(1,1)-t and TARCH(1,1)-t are also doing well. Interestingly, GARCH(1,1)-t is among these better performing models. During the stressed time, the same group of models (EGARCH(1,1)-t, PARCH(1,1)-t, TARCH(1,1)-t, GARCH(1,1)-t) are also the better performing models. This is a quite compelling result that is again showing how flawed the Normal distribution assumption is.
- The asymmetric models do not perform better than the symmetric ones. As Table 3.1 shows, the distributions of the daily log returns are skewed only slightly to the left. In fact, for one of the market, namely HSI, its distribution of log returns has a slight positive skewness.
- RiskMetrics is not the worst among these models. In fact, compared to EGARCH(1,1)-t, it produces the same value for the average of the 99th percentile of the distribution. Nevertheless, EGARCH(1,1)-t outperforms RiskMetrics in terms of number of *exceptions*. This means that the multiplicative factor k will be smaller. As a result, EGARCH(1,1)-t will produce a smaller capital charge. Figure 3.4 shows the comparison between the performances of RiskMetrics and EGARCH(1,1)-t for 2008 out-of-sample VaR forecast of AXKO market index.

3.7 Conclusion

Nine ARCH-type volatility models are estimated for five different stock markets. We estimate the models to forecast 250 daily VaRs in both actual normal and stressed market scenarios. Models that assume t-distribution for the error terms perform better than the ones with assumed Normal distribution. Due to relatively mild skewness in the data, allowing the asymmetry in the models do not improve the performance of the volatility models.

Among these models, EGARCH(1,1) with assumed t-distribution for the error terms turns out to be the best model in tracking the extreme values of the daily log returns. It consistently produces

Table 3.4: AXKO: Backtest of Value at Risk of Mid-2011 to Mid-2012

AXKO 2012	RMSE	95% Value at Risk		99% Value at Risk		
		# of exception	P-Value	# of exception	P-Value	Basel Indicator
EGARCH(1,1)t	0.799%	13	0.885	2	0.742	Green
EGARCH(1,1)	0.795%	16	0.329	4	0.380	Green
PARCH(1,1)t	0.810%	12	0.884	3	0.758	Green
PARCH(1,1)	0.809%	21	0.024	4	0.380	Green
TARCH(1,1)t	0.810%	14	0.669	3	0.758	Green
TARCH(1,1)	0.809%	16	0.329	4	0.380	Green
GARCH(1,1)t	0.811%	11	0.657	3	0.758	Green
GARCH(1,1)	0.810%	21	0.024	5	0.162	Yellow
RiskMetrics	0.818%	15	0.481	5	0.162	Yellow

Table 3.5: HSI: Backtest of Value at Risk of Mid-2011 to Mid-2012

HSI 2012	RMSE	95% Value at Risk		99% Value at Risk		
		# of exception	P-Value	# of exception	P-Value	Basel Indicator
EGARCH(1,1)t	1.115%	11	0.657	3	0.758	Green
EGARCH(1,1)	1.112%	16	0.329	8	0.005	Yellow
PARCH(1,1)t	1.124%	11	0.657	5	0.162	Yellow
PARCH(1,1)	1.122%	18	0.133	8	0.005	Yellow
TARCH(1,1)t	1.122%	12	0.884	3	0.758	Green
TARCH(1,1)	1.118%	14	0.669	8	0.005	Yellow
GARCH(1,1)t	1.125%	12	0.884	4	0.380	Green
GARCH(1,1)	1.123%	17	0.215	8	0.005	Yellow
RiskMetrics	1.140%	15	0.481	8	0.005	Yellow

the smallest number of *exceptions*. Compared to the popular RiskMetrics method, EGARCH(1,1)-t will produce a smaller average capital charge.

Table 3.6: FTSE: Backtest of Value at Risk of Mid-2011 to Mid-2012

FTSE 2012	RMSE	95% Value at Risk		99% Value at Risk		
		# of exception	P-Value	# of exception	P-Value	Basel Indicator
EGARCH(1,1)t	0.869%	15	0.481	3	0.758	Green
EGARCH(1,1)	0.867%	16	0.329	6	0.059	Yellow
PARCH(1,1)t	0.914%	17	0.215	4	0.380	Green
PARCH(1,1)	0.915%	20	0.044	6	0.059	Yellow
TARCH(1,1)t	0.886%	15	0.481	4	0.380	Green
TARCH(1,1)	0.884%	16	0.329	5	0.162	Yellow
GARCH(1,1)t	0.914%	17	0.215	4	0.380	Green
GARCH(1,1)	0.915%	20	0.044	6	0.059	Yellow
RiskMetrics	0.925%	18	0.133	5	0.162	Yellow

Table 3.7: BVSP: Backtest of Value at Risk of Mid-2011 to Mid-2012

BVSP 2012	RMSE	95% Value at Risk		99% Value at Risk		
		# of exception	P-Value	# of exception	P-Value	Basel Indicator
EGARCH(1,1)t	1.168%	7	0.083	3	0.758	Green
EGARCH(1,1)	1.166%	10	0.453	5	0.162	Yellow
PARCH(1,1)t	1.168%	10	0.453	2	0.742	Green
PARCH(1,1)	1.167%	14	0.669	4	0.380	Green
TARCH(1,1)t	1.175%	9	0.286	3	0.758	Green
TARCH(1,1)	1.174%	11	0.657	4	0.380	Green
GARCH(1,1)t	1.165%	11	0.657	2	0.742	Green
GARCH(1,1)	1.165%	14	0.669	4	0.380	Green
RiskMetrics	1.186%	13	0.885	4	0.380	Green

Table 3.8: GSPC: Backtest of Value at Risk of Mid-2011 to Mid-2012

GSPC 2012	RMSE	95% Value at Risk		99% Value at Risk		
		# of exception	P-Value	# of exception	P-Value	Basel Indicator
EGARCH(1,1)t	0.963%	13	0.885	2	0.742	Green
EGARCH(1,1)	0.956%	19	0.079	5	0.162	Yellow
PARCH(1,1)t	1.016%	12	0.884	2	0.742	Green
PARCH(1,1)	1.006%	16	0.329	6	0.059	Yellow
TARCH(1,1)t	0.995%	12	0.884	3	0.758	Green
TARCH(1,1)	0.985%	16	0.329	7	0.019	Yellow
GARCH(1,1)t	1.017%	12	0.884	2	0.742	Green
GARCH(1,1)	1.007%	16	0.329	5	0.162	Yellow
RiskMetrics	1.029%	16	0.329	5	0.162	Yellow

Table 3.9: AXKO: Backtest of Value at Risk of 2008

AXKO 2008	RMSE	95% Value at Risk		99% Value at Risk		
		# of exception	P-Value	# of exception	P-Value	Basel Indicator
EGARCH(1,1)t	1.273%	20	0.044	3	0.758	Green
EGARCH(1,1)	1.269%	26	0.001	10	0.000	Red
PARCH(1,1)t	1.353%	13	0.885	3	0.758	Green
PARCH(1,1)	1.348%	19	0.079	6	0.059	Yellow
TARCH(1,1)t	1.337%	14	0.669	2	0.742	Green
TARCH(1,1)	1.332%	20	0.044	6	0.059	Yellow
GARCH(1,1)t	1.352%	13	0.885	3	0.758	Green
GARCH(1,1)	1.347%	19	0.079	7	0.019	Yellow
RiskMetrics	1.389%	15	0.481	5	0.162	Yellow

Table 3.10: HSI: Backtest of Value at Risk of 2008o

HSI 2008	RMSE	95% Value at Risk		99% Value at Risk		
		# of exception	P-Value	# of exception	P-Value	Basel Indicator
EGARCH(1,1)t	2.068%	16	0.329	3	0.758	Green
EGARCH(1,1)	2.061%	24	0.003	7	0.019	Yellow
PARCH(1,1)t	2.212%	14	0.669	4	0.380	Green
PARCH(1,1)	2.184%	17	0.215	9	0.001	Yellow
TARCH(1,1)t	2.167%	13	0.885	2	0.742	Green
TARCH(1,1)	2.145%	18	0.133	7	0.019	Yellow
GARCH(1,1)t	2.231%	13	0.885	4	0.380	Green
GARCH(1,1)	2.227%	15	0.481	8	0.005	Yellow
RiskMetrics	2.274%	14	0.669	7	0.019	Yellow

Table 3.11: BVSP: Backtest of Value at Risk of 2008

BVSP 2008	RMSE	95% Value at Risk		99% Value at Risk		
		# of exception	P-Value	# of exception	P-Value	Basel Indicator
EGARCH(1,1)t	2.086%	23	0.006	7	0.019	Yellow
EGARCH(1,1)	2.092%	28	0.000	9	0.001	Yellow
PARCH(1,1)t	2.173%	19	0.079	3	0.758	Green
PARCH(1,1)	2.162%	25	0.001	8	0.005	Yellow
TARCH(1,1)t	2.075%	18	0.133	4	0.380	Green
TARCH(1,1)	2.072%	27	0.000	6	0.059	Yellow
GARCH(1,1)t	2.158%	21	0.024	3	0.758	Green
GARCH(1,1)	2.145%	26	0.001	8	0.005	Yellow
RiskMetrics	2.277%	19	0.079	4	0.380	Green

Table 3.12: FTSE: Backtest of Value at Risk of 2008

FTSE 2008	RMSE	95% Value at Risk		99% Value at Risk		
		# of exception	P-Value	# of exception	P-Value	Basel Indicator
EGARCH(1,1)t	0.961%	8	0.853	3	0.279	Green
EGARCH(1,1)	0.960%	8	0.853	4	0.089	Green
PARCH(1,1)t	1.030%	10	0.372	5	0.024	Yellow
PARCH(1,1)	1.026%	10	0.372	5	0.024	Yellow
TARCH(1,1)t	0.965%	8	0.853	4	0.089	Green
TARCH(1,1)	0.964%	8	0.853	4	0.089	Green
GARCH(1,1)t	1.028%	10	0.372	5	0.024	Yellow
GARCH(1,1)	1.027%	10	0.372	5	0.024	Yellow
RiskMetrics	1.048%	10	0.372	5	0.024	Yellow

Table 3.13: GSPC: Backtest of Value at Risk of 2008

GSPC 2008	RMSE	95% Value at Risk		99% Value at Risk		
		# of exception	P-Value	# of exception	P-Value	Basel Indicator
EGARCH(1,1)t	1.593%	21	0.024	6	0.059	Yellow
EGARCH(1,1)	1.589%	29	0.000	9	0.001	Yellow
PARCH(1,1)t	1.695%	18	0.133	4	0.380	Green
PARCH(1,1)	1.679%	24	0.003	9	0.001	Yellow
TARCH(1,1)t	1.667%	18	0.133	3	0.758	Green
TARCH(1,1)	1.654%	21	0.024	6	0.059	Yellow
GARCH(1,1)t	1.694%	18	0.133	4	0.380	Green
GARCH(1,1)	1.674%	24	0.003	10	0.000	Red
RiskMetrics	1.714%	20	0.044	7	0.019	Yellow

Table 3.14: AXKO: Volatility Model Estimation Results Using Daily Log Return, August 2000 - June 2011 (2737 trading days)

	(1) RiskMetrics	(2) GARCH(1,1)	(3) GARCH(1,1)t	(4) TARCH(1,1)	(5) TARCH(1,1)t
AXKO					
d2008	-0.00 (-1.05)	-0.00 (-1.68)	-0.00 (-2.63)**	-0.00 (-1.19)	-0.00 (-1.40)
_cons	0.00 .	0.00 (3.95)***	0.00 (4.43)***	0.00 (2.00)*	0.00 (2.84)**
ARCH					
L.arch	0.09 (11.96)***	0.09 (11.93)***	0.08 (8.05)***	0.15 (11.96)***	0.14 (9.43)***
L.garch	0.91 (127.22)***	0.91 (117.84)***	0.92 (92.98)***	0.91 (118.45)***	0.92 (98.11)***
L.tarch				-0.14 (-10.69)***	-0.13 (-7.85)***
_cons	0.00 (4.68)***	0.00 (3.87)***	0.00 (2.81)**	0.00 (5.71)***	0.00 (4.28)***
Indfm2					
_cons			1.94 (10.12)***		2.30 (9.76)***
<i>N</i>	2737	2737	2737	2737	2737
Log Likelihood	9199.15	9207.82	9240.80	9256.01	9273.02
AIC	-18392.30	-18405.64	-18469.59	-18500.02	-18532.03
BIC	-18374.55	-18376.06	-18434.11	-18464.53	-18490.63

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 3.15: AXKO: Volatility Model Estimation Results Using Daily Log Return, August 2000 - June 2011 (2737 trading days)

	(1)	(2)	(3)	(4)
	PARCH(1,1)	PARCH(1,1)t	EGARCH(1,1)	EGARCH(1,1)t
AXKO				
d2008	-0.00 (-1.75)	-0.00 (-2.67)**	-0.00 (-0.75)	-0.00 (-1.07)
_cons	0.00 (3.98)***	0.00 (4.45)***	0.00 (1.66)	0.00 (2.61)**
ARCH				
L.PARCH	0.08 (7.85)***	0.08 (5.54)***		
L.pgarch	0.91 (114.68)***	0.92 (91.49)***		
L.earch			-0.12 (-12.56)***	-0.11 (-9.06)***
L.earch_a			0.15 (10.92)***	0.14 (8.21)***
L.egarch			0.98 (350.48)***	0.98 (317.31)***
_cons	0.00 (0.80)	0.00 (0.53)	-0.18 (-6.54)***	-0.14 (-4.86)***
POWER				
power	2.26 (9.35)***	2.19 (5.96)***		
Indfm2				
_cons		1.94 (10.07)***		2.35 (9.65)***
<i>N</i>	2737	2737	2737	2737
Log Likelihood	9208.18	9240.92	9259.71	9275.15
AIC	-18404.37	-18467.83	-18507.42	-18536.30
BIC	-18368.88	-18426.43	-18471.93	-18494.89

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 3.16: HSI: Volatility Model Estimation Results Using Daily Log Return, August 2000 - June 2011 (2714 trading days)

	(1) RiskMetrics	(2) GARCH(1,1)	(3) GARCH(1,1)t	(4) TARCH(1,1)	(5) TARCH(1,1)t
HSI					
d2008	-0.00 (-1.30)	-0.00 (-1.81)	-0.00 (-2.12)*	-0.00 (-1.49)	-0.00 (-1.68)
_cons	0.00 .	0.00 (2.42)*	0.00 (2.65)**	0.00 (1.13)	0.00 (1.73)
ARCH					
L.arch	0.07 (9.76)***	0.07 (9.81)***	0.06 (7.16)***	0.10 (10.10)***	0.10 (7.97)***
L.garch	0.93 (128.39)***	0.93 (119.93)***	0.94 (108.12)***	0.92 (108.09)***	0.93 (99.04)***
L.tarch				-0.08 (-7.92)***	-0.08 (-5.65)***
_cons	0.00 (3.90)***	0.00 (3.55)***	0.00 (2.55)*	0.00 (5.29)***	0.00 (3.80)***
Indfm2					
_cons			2.00 (9.02)***		2.15 (8.93)***
N	2714	2714	2714	2714	2714
Log Likelihood	7917.68	7921.99	7945.34	7942.61	7960.18
AIC	-15829.36	-15833.98	-15878.69	-15873.22	-15906.37
BIC	-15811.64	-15804.44	-15843.25	-15837.79	-15865.03

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 3.17: HSI: Volatility Model Estimation Results Using Daily Log Return, August 2000 - June 2011 (2714 trading days)

	(1)	(2)	(3)	(4)
	PARCH(1,1)	PARCH(1,1)t	EGARCH(1,1)	EGARCH(1,1)t
HSI				
d2008	-0.00 (-1.67)	-0.00 (-2.05)*	-0.00 (-1.11)	
_cons	0.00 (2.56)*	0.00 (2.79)**	0.00 (1.33)	0.00 (1.87)
ARCH				
L.PARCH	0.08 (10.00)***	0.07 (7.23)***		
L.pgarch	0.93 (122.68)***	0.94 (110.69)***		
L.earch			-0.06 (-7.50)***	-0.06 (-5.89)***
L.earch_a			0.13 (8.96)***	0.13 (7.69)***
L.egarch			0.99 (412.22)***	0.99 (357.03)***
_cons	0.00 (0.97)	0.00 (0.66)	-0.11 (-5.49)***	-0.10 (-3.97)***
POWER				
power	1.26 (6.20)***	1.33 (4.34)***		
Indfm2				
_cons		2.02 (8.88)***		2.12 (8.92)***
<i>N</i>	2714	2714	2714	2714
Log Likelihood	7924.74	7946.71	7943.42	7961.31
AIC	-15837.48	-15879.42	-15874.85	-15910.62
BIC	-15802.04	-15838.08	-15839.41	-15875.18

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 3.18: FTSE: Volatility Model Estimation Results Using Daily Log Return, August 2000 - June 2011 (2749 trading days)

	(1) RiskMetrics	(2) GARCH(1,1)	(3) GARCH(1,1)t	(4) TARCH(1,1)	(5) TARCH(1,1)t
FTSE					
d2008	-0.00 (-0.98)	-0.00 (-1.51)	-0.00 (-1.58)	0.00 (0.92)	0.00 (0.29)
_cons	0.00 .	0.00 (2.31)*	0.00 (2.84)**	-0.00 (-0.23)	0.00 (0.43)
ARCH					
L.arch	0.11 (11.42)***	0.10 (10.62)***	0.10 (8.71)***	0.15 (12.34)***	0.16 (10.44)***
L.garch	0.89 (95.30)***	0.89 (94.41)***	0.89 (79.58)***	0.92 (110.10)***	0.92 (96.11)***
L.tarch				-0.17 (-11.59)***	-0.17 (-9.70)***
_cons	0.00 (4.36)***	0.00 (3.89)***	0.00 (3.27)**	0.00 (6.76)***	0.00 (5.46)***
Indfm2					
_cons			2.46 (8.48)***		2.77 (7.96)***
N	2749	2749	2749	2749	2749
Log Likelihood	8659.15	8662.59	8673.79	8715.61	8722.48
AIC	-17312.29	-17315.18	-17335.58	-17419.23	-17430.95
BIC	-17294.53	-17285.59	-17300.07	-17383.71	-17389.52

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 3.19: FTSE: Volatility Model Estimation Results Using Daily Log Return, August 2000 - June 2011 (2749 trading days)

	(1)	(2)	(3)	(4)
	PARCH(1,1)	PARCH(1,1)t	EGARCH(1,1)	EGARCH(1,1)t
FTSE				
d2008	-0.00 (-1.52)	-0.00 (-1.58)	0.00 (1.29)	0.00 (0.45)
_cons	0.00 (2.31)*	0.00 (2.84)**	-0.00 (-0.36)	0.00 (0.23)
ARCH				
L.PARCH	0.11 (9.34)***	0.10 (7.27)***		
L.pgarch	0.89 (88.53)***	0.89 (74.40)***		
L.earch			-0.13 (-12.81)***	-0.13 (-11.42)***
L.earch_a			0.11 (7.69)***	0.11 (6.72)***
L.egarch			0.98 (425.13)***	0.98 (381.08)***
_cons	0.00 (0.79)	0.00 (0.59)	-0.15 (-7.02)***	-0.14 (-6.00)***
POWER				
power	1.94 (7.07)***	1.98 (5.47)***		
Indfm2				
_cons		2.46 (8.48)***		2.92 (7.56)***
<i>N</i>	2749	2749	2749	2749
Log Likelihood	8662.60	8673.79	8725.19	8730.34
AIC	-17313.20	-17333.58	-17438.38	-17446.69
BIC	-17277.69	-17292.15	-17402.87	-17405.25

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 3.20: BVSP: Volatility Model Estimation Results Using Daily Log Return, August 2000 - June 2011 (2687 trading days)

	(1) RiskMetrics	(2) GARCH(1,1)	(3) GARCH(1,1)t	(4) TARCH(1,1)	(5) TARCH(1,1)t
BVSP					
d2008	-0.00 (-0.81)	-0.00 (-1.55)	-0.00 (-1.68)	-0.00 (-0.50)	-0.00 (-0.79)
_cons	0.00 .	0.00 (2.76)**	0.00 (3.47)***	0.00 (1.48)	0.00 (2.28)*
ARCH					
L.arch	0.07 (9.62)***	0.07 (8.73)***	0.07 (6.78)***	0.11 (8.58)***	0.11 (7.21)***
L.garch	0.93 (126.50)***	0.91 (93.15)***	0.92 (76.41)***	0.91 (82.43)***	0.91 (69.83)***
L.tarch				-0.10 (-6.98)***	-0.10 (-5.81)***
_cons	0.00 (3.84)***	0.00 (4.32)***	0.00 (3.16)**	0.00 (5.64)***	0.00 (4.35)***
Indfm2					
_cons			2.24 (9.50)***		2.42 (9.67)***
N	2687	2687	2687	2687	2687
Log Likelihood	7060.50	7070.43	7089.07	7094.58	7108.75
AIC	-14114.99	-14130.85	-14166.14	-14177.17	-14203.49
BIC	-14097.30	-14101.37	-14130.76	-14141.79	-14162.22

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 3.21: BVSP: Volatility Model Estimation Results Using Daily Log Return, August 2000 - June 2011 (2687 trading days)

	(1)	(2)	(3)	(4)
	PARCH(1,1)	PARCH(1,1)t	EGARCH(1,1)	EGARCH(1,1)t
BVSP				
d2008	-0.00 (-1.51)	-0.00 (-1.66)	-0.00 (-0.18)	-0.00 (-0.53)
_cons	0.00 (2.71)**	0.00 (3.44)***	0.00 (1.39)	0.00 (2.20)*
ARCH				
L.PARCH	0.06 (4.97)***	0.06 (3.71)***		
L.pgarch	0.91 (87.83)***	0.91 (71.26)***		
L.earch			-0.08 (-8.30)***	-0.08 (-6.71)***
L.earch_a			0.13 (8.28)***	0.13 (6.75)***
L.egarch			0.98 (242.78)***	0.98 (203.87)***
_cons	0.00 (0.57)	0.00 (0.44)	-0.19 (-5.93)***	-0.18 (-4.50)***
POWER				
power	2.40 (5.76)***	2.49 (4.60)***		
Indfm2				
_cons		2.23 (9.48)***		2.41 (9.82)***
<i>N</i>	2687	2687	2687	2687
Log Likelihood	7070.86	7089.57	7092.47	7107.18
AIC	-14129.73	-14165.15	-14172.94	-14200.36
BIC	-14094.35	-14123.87	-14137.56	-14159.08

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 3.22: GSPC: Volatility Model Estimation Results Using Daily Log Return, August 2000 - June 2011 (2739 trading days)

	(1) RiskMetrics	(2) GARCH(1,1)	(3) GARCH(1,1)t	(4) TARCH(1,1)	(5) TARCH(1,1)t
GSPC					
d2008	-0.00 (-1.57)	-0.00 (-2.21)*	-0.00 (-1.70)	0.00 (0.03)	-0.00 (-0.40)
_cons	0.00 .	0.00 (2.41)*	0.00 (3.28)**	-0.00 (-0.28)	0.00 (1.09)
ARCH					
L.arch	0.09 (10.40)***	0.08 (10.07)***	0.08 (7.78)***	0.12 (11.40)***	0.12 (9.53)***
L.garch	0.91 (108.84)***	0.91 (105.36)***	0.91 (89.42)***	0.94 (145.01)***	0.94 (124.47)***
L.tarch				-0.14 (-12.44)***	-0.15 (-10.06)***
_cons	0.00 (5.82)***	0.00 (6.14)***	0.00 (2.91)**	0.00 (7.86)***	0.00 (4.87)***
Indfm2					
_cons			1.88 (9.13)***		2.14 (9.86)***
<i>N</i>	2739	2739	2739	2739	2739
Log Likelihood	8572.39	8577.99	8608.25	8637.89	8658.43
AIC	-17138.78	-17145.98	-17204.50	-17263.78	-17302.86
BIC	-17121.03	-17116.40	-17169.00	-17228.29	-17261.46

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 3.23: GSPC: Volatility Model Estimation Results Using Daily Log Return, August 2000 - June 2011 (2739 trading days)

	(1)	(2)	(3)	(4)
	PARCH(1,1)	PARCH(1,1)t	EGARCH(1,1)	EGARCH(1,1)t
GSPC				
d2008	-0.00 (-2.03)*	-0.00 (-1.62)	-0.00 (-0.02)	-0.00 (-0.43)
_cons	0.00 (2.40)*	0.00 (3.29)**	0.00 (0.08)	0.00 (1.48)
ARCH				
L.parch	0.07 (6.12)***	0.08 (5.24)***		
L.pgarch	0.91 (97.14)***	0.91 (83.26)***		
L.earch			-0.12 (-12.82)***	-0.12 (-10.46)***
L.earch_a			0.10 (8.12)***	0.09 (6.20)***
L.egarch			0.98 (519.35)***	0.99 (434.41)***
_cons	0.00 (0.52)	0.00 (0.45)	-0.15 (-8.80)***	-0.11 (-5.43)***
POWER				
power	2.41 (6.19)***	2.23 (4.90)***		
lndfm2				
_cons		1.88 (9.06)***		1.98 (10.05)***
<i>N</i>	2739	2739	2739	2739
Log Likelihood	8578.40	8608.38	8626.84	8655.68
AIC	-17144.80	-17202.75	-17241.69	-17297.36
BIC	-17109.31	-17161.34	-17206.19	-17255.95

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

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Appendix A

Computer Codes

A.1 Parts of Matlab Code Used in Chapter 1

```
% est.m: Maximizes the (minimizes the negative) log likelihood function for
% the New Keynesian model with markup and technology shocks.
% When maximizing the log likelihood function, the parameters are
% transformed to satisfy theoretical restrictions. The log
% likelihood function with transformed parameters is contained in
% llfn.m.
% THIS PROGRAM WAS WRITTEN FOR MATLAB BY
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% NOTICE.

% load data and choose sample

global gt pt rt

load gpr.dat;

% gt = gpr(:,1);

% pt = gpr(:,2);

% rt = gpr(:,3);

% gt = gpr(1:127,1);

% pt = gpr(1:127,2);

% rt = gpr(1:127,3);

gt = gpr(128:220,1);

pt = gpr(128:220,2);

rt = gpr(128:220,3);

ztr = 0.0048;

ptr = 0.0086;

betatr = sqrt(0.99/0.01)/100;

omegatr = sqrt(0.625/0.375);

```

psitr = 0.10;
alphaxtr = sqrt(0.25/0.75);
alphaptr = sqrt(0.25/0.75);
rhortr = 1;
rhoPtr = 0.6017;
rhoGtr = 0.4240;
rhoXtr = 0.0770;
rhoatr = 0.0205;
rhoetr = 0.0882;
sigatr = 0.0150;
sigetr = 0.0008;
sigztr = 0.0000;
sigrtr = 0.0030;
bigtheto = [ omegatr alphaxtr alphaptr ...
rhoPtr rhoGtr rhoXtr ...
rhoatr rhoetr ...
sigatr sigetr sigztr sigrtr ]';

```

```

% maximize likelihood

options = optimset('Display','iter','LargeScale','off','MaxFunEvals',2500,'MaxIter',2500);
thetstar = fminunc(@llfn,bigtheto,options);

function llfn = llfn(bigthet);

% Uses the Kalman filter to evaluate the negative log likelihood function
% for the New Keynesian model with markup and technology shocks. The
% parameters are transformed to satisfy theoretical restrictions.
%

% define variables and parameters

global gt pt rt
capt = length(gt);
bigthet = real(bigthet);

ztr = 0.0048;
ptr = 0.0086;
betatr = sqrt(0.99/0.01)/100;
omegatr = sqrt(0.625/0.375);
psitr = 0.10;
alphaxtr = sqrt(0.25/0.75);
alphaptr = sqrt(0.25/0.75);
rhortr = 1;
rhoPtr = 0.25;
rhogtr = 0.05;
rhoxtr = 0.05;
rhoatr = 0;
rhoetr = 0;
sigatr = 0.1;
sigetr = 0.1;

```

```

sigztr = 0.1;
sigrtr = 0.1;
omegatr = bigthet(1);
alphaxtr = bigthet(2);
alphaptr = bigthet(3);
rhoptr = bigthet(4);
rhogtr = bigthet(5);
rhoxtr = bigthet(6);
rhoatr = bigthet(7);
rhoetr = bigthet(8);
sigatr = bigthet(9);
sigetr = bigthet(10);
sigztr = bigthet(11);
sigrtr = bigthet(12);
% untransform parameters
z = 1 + abs(ztr);
p = 1 + abs(ptr);
beta = (100*betatr)^2/(1+(100*betatr)^2);
omega = omegatr^2/(1+omegatr^2);
psi = abs(psitr);
alphax = alphaxtr^2/(1+alphaxtr^2);
alphap = alphaptr^2/(1+alphaptr^2);
rhor = abs(rhotr);
rhop = abs(rhoptr);
rhog = abs(rhogtr);
rhox = abs(rhoxtr);
rhoa = (100*rhoatr)^2/(1+(100*rhoatr)^2);

```



```

rhoe = (100*rhoetr)^2/(1+(100*rhoetr)^2);
siga = abs(sigatr);
sige = abs(sigetr);
sigz = abs(sigztr);
sigr = abs(sigrtr);
% find steady state
gss = z;
pss = p;
rss = pss*(z/beta);
% form matrices A, B, and C
biga = [ 0 -1 0 0 0 1 1-alphax ; ...
0 0 0 0 psi beta*(1-alphap) 0 ; ...
-1 0 0 1 0 0 0 ; ...
1 0 0 0 0 0 0 ; ...
0 1 -rhop -rhog -rhox 0 0 ; ...
0 0 1 0 0 0 0 ; ...
0 0 0 0 1 0 0 ];
bigb = [ 0 0 0 0 -alphax 0 1 ; ...
0 0 -beta*alphap 0 0 1 0 ; ...
-1 0 0 0 0 0 0 ; ...
0 0 0 0 0 0 1 ; ...
0 rhor 0 0 0 0 0 ; ...
0 0 0 0 0 1 0 ; ...
0 0 0 0 0 0 1 ];
bigc = [ (omega-1)*(1-rhoa) 0 0 0 ; ...
0 1 0 0 ; ...
0 0 1 0 ; ...

```

```

omega 0 0 0 ; ...
0 0 0 1 ; ...
0 0 0 0 ;
0 0 0 0 ];

% form matrix P
bigp = [ rhoa 0 0 0 ; ...
0 rhoe 0 0 ; ...
0 0 0 0 ; ...
0 0 0 0 ];

% form matrices Q, Z, S, and T
[bigq,bigt,bigz] = qz(biga,bigb);
[bigq,bigt,bigz] = reorder(bigt,bigq,bigz);
bigq1 = bigq(1:5,:);
bigq2 = bigq(6:7,:);
bigz11 = bigz(1:5,1:5);
bigz12 = bigz(1:5,6:7);
bigz21 = bigz(6:7,1:5);
bigz22 = bigz(6:7,6:7);
bigq11 = bigq1(1:5,1:5);
bigq12 = bigq1(1:5,6:7);
bigq21 = bigq2(6:7,1:5);
bigq22 = bigq2(6:7,6:7);
bigt11 = bigt(1:5,1:5);
bigt12 = bigt(1:5,6:7);
bigt21 = bigt(6:7,1:5);
bigt22 = bigt(6:7,6:7);
lamviol = 0;
if abs(bigt11(5,5)/bigt11(5,5)) > 1
lamviol = 1;

```

```

end
if abs(bigt22(1,1)/bigs22(1,1)) < 1
lamviol = 1;
end
% form matrix R
bigra = bigs22*inv(bigt22);
bigrb = bigq2*bigc;
vecr = inv(eye(8)-kron(bigr,bigra))*bigrb(:);
bigr = reshape(vecr,2,4);
% form matrices M
bigm3 = bigz11*inv(bigs11)*bigt11*inv(bigz11);
bigm4a = bigt11*inv(bigz11)*bigz12*inv(bigt22)*bigr ...
+ bigq1*bigc + bigs12*inv(bigt22)*bigr*bigp ...
- bigt12*inv(bigt22)*bigr;
bigm4 = bigz11*inv(bigs11)*bigm4a - bigz12*inv(bigt22)*bigr*bigp;
% form matrices PI and W
bigpi = [ bigm3 bigm4 ; zeros(4,5) bigp ];
bigw = [ zeros(5,4) ; eye(4) ];
bigpi = real(bigpi);
% form matrices AX, BX, CX, VX, and BVBX
bigax = bigpi;
bigbx = bigw;
bigcx = [ bigpi(4,:) ; bigpi(3,:) ; bigpi(2,:) ];
bigvx = diag([siga^2 sige^2 sigz^2 sigr^2]);
bigvbvx = bigbx*bigvx*bigbx';
% put data in deviation form
% gthat = gt - log(gss);

```

```

% pthat = pt - log(pss);
% rthat = rt - log(rss);
gthat = gt - mean(gt);
pthat = pt - mean(pt);
rthat = rt - mean(rt);
dthat = [ gthat pthat rthat ];
% evaluate negative log likelihood
st = zeros(9,1);
bigsig1 = inv(eye(81)-kron(bigax,bigax))*bigbvbvbx(:);
bigsig1 = reshape(bigsig1,9,9);
llfn = (3*capt/2)*log(2*pi);
for t = 1:capt
    ut = dthat(t,:) - bigcx*st;
    omegt = bigcx*bigsig1*bigcx';
    omeginvt = inv(omegt);
    llfn = llfn + (1/2)*(log(det(omegt))+ut'*omeginvt*ut);
    bigkt = bigax*bigsig1*bigcx'*omeginvt;
    st = bigax*st + bigkt*ut;
    bigsig1 = bigbvbvbx + bigax*bigsig1*bigax' ...
    - bigax*bigsig1*bigcx'*omeginvt*bigcx*bigsig1*bigax';
end
% penalize eigenvalue constraint violations
if lamviol
    llfn = llfn + 1e12;
end
if abs(imag(llfn)) > 0
    llfn = real(llfn) + 1e12;
end

```

end

A.2 Parts of Matlab Code Used in Chapter 2

```
n1 = 8 ;% Number of predetermined variables (nx)
n2 = 8;% Number of non-predetermined variables (ny)
n3 = [3,4,5]; % Position of observable in non-predetermined vector [pie,r,y]
nyo=size(n3,2);
%delta1=prctile(param(:,1:25),50,1)';
delta1=mean(param(:,1:25))';
[retcode,gx,hx,gxx,hxx,gss,hss,eta] = AT1_modsol_Klein1(order,delta1,n1,n2,n3);
nx=size(eta,1);
nx2=size(eta,2);
nx1=nx-nx2;
if retcode==0
deltaviolate=[deltaviolate delta1];
invcondviol=invcondviol+1;
u=0;
else
sigmav=delta1(end-nyo+1:end);
omegav=diag(eta(nx1+1:end,:));
meanx0=zeros(n1,1);
meanx0=repmat(meanx0,1,N);
[sigyy0,sigxx0,sigxy0,sigyx0]=mom(gx,hx,eta*eta',0);
x0=meanx0+(chol(sigxx0)')*randn(n1,N);
[x1,xm,neff,lnlv] = AT1LGCPF_KKSS(x0,gx,hx,gxx(n3,:),hxx,gss(n3),hss,omegav,sigmav,Yo,n3,N);
end
```

```

lnl1=sum(lnlv);
ym1=repmat((.5*gss),1,T)+gx*xm+.5*gxx*kronmc(xm);
ym2=ym1';
ym=ym2(:,3:5);
figure(1)
plot(Yo);
hold;
plot(ym);
RMSE=sqrt(mean((Yo-ym).*(Yo-ym)));
MAPE=mean(abs((Yo-ym)./Yo));
figure(2)
xm1=xm';
infcomp=zeros(T,3);
infcomp(:,1)=Yo(:,1);
infcomp(:,2)=ym2(:,3);
infcomp(:,3)=xm1(:,6);
plot(infcomp);
load matlab_120q_N20k_M50k.mat;
fig_str2=cellstr(strvcat('Tech persistence','Lab disutility',...
'Relative Risk Aversion',...
'Pi in Taylor Rule','Calvo parameter','Theta','Iota',...
'Inflation target persistence','Tax persistence'));
fig_str3=cellstr(strvcat('Habit persistence',...
'Interest rate persistence in Taylor Rule',...
'SE of inflation target shock',...
'SE of Tech shock','SE of tax shock',...
'Output coeff in Taylor Rule','Target mean',...

```

```

'SE of monetary policy shock','Tax shock mean'));
paramchart=zeros(M,npar-7);
paramchart(:,1)=param(:,1);
paramchart(:,2)=param(:,3);
for i=3:6
paramchart(:,i)=param(:,i+2);
end
for i=7:16
paramchart(:,i)=param(:,i+3);
end
paramchart(:,17)=param(:,21);
paramchart(:,18)=param(:,22);
priorparamch=zeros(M,npar-7);
priorparamch(:,1)=priorparam(:,1);
priorparamch(:,2)=priorparam(:,3);
for i=3:6
priorparamch(:,i)=priorparam(:,i+2);
end
for i=7:16
priorparamch(:,i)=priorparam(:,i+3);
end
priorparamch(:,17)=priorparam(:,21);
priorparamch(:,18)=priorparam(:,22);
charcol=size(fig_str2,1);
rows_graph=3;
cols_graph=3;
figure(1);

```

```

for ip=1:charcol
subplot(rows_graph,cols_graph,ip);
hist([paramchart(:,ip) priorparamch(:,ip)],M/50);
title(fig_str2(ip));
end
figure(2);
for ip=1:charcol
subplot(rows_graph,cols_graph,ip);
hist([paramchart(:,ip+9) priorparamch(:,ip+9)],M/50);
title(fig_str3(ip));
end

```



```

% First, run Calculating_xm.m

tic;

M=size(solm,1);

n1=8;

n2=8;

n3=[3 4 5];

ne=4; % number of shocks

nh=11; %max horizon for IRF computations

%x0m1=[0.003509751 0.053030985 0.095359257 0.006902211...
%0.011753731 0.040389977 0.000413504 -0.002703563];

%x0m2=[0.003053491 0.053338442 0.042499326 0.021234911...
% 0.043541823 0.02865939 0.000448817 0.005569858];

% x0m=xm(:,[31 118]);

% [31 118] is based on data version 1 (144q, all HP-filtered)

x0m=xm(:,[30 55]);

% [30 55] is based on data version 2 (120q, quarterly changes,
% Y detrended with mean)

% x0m is an (n1 x ns) each column a separate starting point

% ns is the number of starting points

shockm=[1 0 0 0;

0 1 0 0;

0 0 1 0;

0 0 0 1];

% shockm is an (ne x ne) matrix with shocks (each column is a separate
% scenario for shocks

dyma=DSGE_IRF_INPUTED_SHOCKS(solm,n1,n2,n3,ne,nh,x0m,shockm);

% OUTPUTS:

```

```

% dyma = (nyo x (nh+1) x ne x ns x M) array with impulse responses
% in this simple example, (3 x 13 x 4 x 1 x 40000)
% STACKING
% With high inflation starting point
% high_impulse11: tech shock (stdtecsh) on pi
% high_impulse12: tech shock (stdtecsh) on int
% high_impulse13: tech shock (stdtecsh) on y
high_impulse11=zeros(M,nh+1);
for ki=1:M
high_impulse11(ki,:)=dyma(1,:,1,1,ki);
end
high_impulsepct11=prctile(high_impulse11,[2.5 50 97.5],1)';
high_impulse12=zeros(M,nh+1);
for ki=1:M
high_impulse12(ki,:)=dyma(2,:,1,1,ki);
end
high_impulsepct12=prctile(high_impulse12,[2.5 50 97.5],1)';
high_impulse13=zeros(M,nh+1);
for ki=1:M
high_impulse13(ki,:)=dyma(3,:,1,1,ki);
end
high_impulsepct13=prctile(high_impulse13,[2.5 50 97.5],1)';
% high_impulse21: target shock (stdmonsh) on pi
% high_impulse22: target shock (stdmonsh) on int
% high_impulse23: target shock (stdmonsh) on y
high_impulse21=zeros(M,nh+1);
for ki=1:M

```

```

high_impulse21(ki,:)=dyma(1,:,2,1,ki);
end
high_impulsepct21=prctile(high_impulse21,[2.5 50 97.5],1)';
high_impulse22=zeros(M,nh+1);
for ki=1:M
high_impulse22(ki,:)=dyma(2,:,2,1,ki);
end
high_impulsepct22=prctile(high_impulse22,[2.5 50 97.5],1)';
high_impulse23=zeros(M,nh+1);
for ki=1:M
high_impulse23(ki,:)=dyma(3,:,2,1,ki);
end
high_impulsepct23=prctile(high_impulse23,[2.5 50 97.5],1)';
% high_impulse31: tax shock (stdtau) on pi
% high_impulse32: tax shock (stdtau) on int
% high_impulse33: tax shock (stdtau) on y
high_impulse31=zeros(M,nh+1);
for ki=1:M
high_impulse31(ki,:)=dyma(1,:,3,1,ki);
end
high_impulsepct31=prctile(high_impulse31,[2.5 50 97.5],1)';
high_impulse32=zeros(M,nh+1);
for ki=1:M
high_impulse32(ki,:)=dyma(2,:,3,1,ki);
end
high_impulsepct32=prctile(high_impulse32,[2.5 50 97.5],1)';
high_impulse33=zeros(M,nh+1);

```

```

for ki=1:M
high_impulse33(ki,:)=dyma(3, :, 3, 1, ki);
end
high_impulsepct33=prctile(high_impulse33,[2.5 50 97.5],1)';
% high_impulse41: monetary policy (stdpol) on pi
% high_impulse42: monetary policy (stdpol) on int
% high_impulse43: monetary policy (stdpol) on y
high_impulse41=zeros(M,nh+1);
for ki=1:M
high_impulse41(ki,:)=dyma(1, :, 4, 1, ki);
end
high_impulsepct41=prctile(high_impulse41,[2.5 50 97.5],1)';
high_impulse42=zeros(M,nh+1);
for ki=1:M
high_impulse42(ki,:)=dyma(2, :, 4, 1, ki);
end
high_impulsepct42=prctile(high_impulse42,[2.5 50 97.5],1)';
high_impulse43=zeros(M,nh+1);
for ki=1:M
high_impulse43(ki,:)=dyma(3, :, 4, 1, ki);
end
high_impulsepct43=prctile(high_impulse43,[2.5 50 97.5],1)';
% With low inflation starting point
% low_impulse11: tech shock (stdtecsh) on pi
% low_impulse12: tech shock (stdtecsh) on int
% low_impulse13: tech shock (stdtecsh) on y
low_impulse11=zeros(M,nh+1);

```

```

for ki=1:M
low_impulse11(ki,:)=dyma(1,:,1,2,ki);
end
low_impulsepct11=prctile(low_impulse11,[2.5 50 97.5],1)';
low_impulse12=zeros(M,nh+1);
for ki=1:M
low_impulse12(ki,:)=dyma(2,:,1,2,ki);
end
low_impulsepct12=prctile(low_impulse12,[2.5 50 97.5],1)';
low_impulse13=zeros(M,nh+1);
for ki=1:M
low_impulse13(ki,:)=dyma(3,:,1,2,ki);
end
low_impulsepct13=prctile(low_impulse13,[2.5 50 97.5],1)';
% low_impulse21: target shock (stdmonsh) on pi
% low_impulse22: target shock (stdmonsh) on int
% low_impulse23: target shock (stdmonsh) on y
low_impulse21=zeros(M,nh+1);
for ki=1:M
low_impulse21(ki,:)=dyma(1,:,2,2,ki);
end
low_impulsepct21=prctile(low_impulse21,[2.5 50 97.5],1)';
low_impulse22=zeros(M,nh+1);
for ki=1:M
low_impulse22(ki,:)=dyma(2,:,2,2,ki);
end
low_impulsepct22=prctile(low_impulse22,[2.5 50 97.5],1)';

```

```

low_impulse23=zeros(M,nh+1);
for ki=1:M
low_impulse23(ki,:)=dyma(3,.,2,2,ki);
end
low_impulsepct23=prctile(low_impulse23,[2.5 50 97.5],1)';
% low_impulse31: tax shock (stdtau) on pi
% low_impulse32: tax shock (stdtau) on int
% low_impulse33: tax shock (stdtau) on y
low_impulse31=zeros(M,nh+1);
for ki=1:M
low_impulse31(ki,:)=dyma(1,.,3,2,ki);
end
low_impulsepct31=prctile(low_impulse31,[2.5 50 97.5],1)';
low_impulse32=zeros(M,nh+1);
for ki=1:M
low_impulse32(ki,:)=dyma(2,.,3,2,ki);
end
low_impulsepct32=prctile(low_impulse32,[2.5 50 97.5],1)';
low_impulse33=zeros(M,nh+1);
for ki=1:M
low_impulse33(ki,:)=dyma(3,.,3,2,ki);
end
low_impulsepct33=prctile(low_impulse33,[2.5 50 97.5],1)';
% low_impulse41: monetary policy (stdpol) on pi
% low_impulse42: monetary policy (stdpol) on int
% low_impulse43: monetary policy (stdpol) on y
low_impulse41=zeros(M,nh+1);

```

```

for ki=1:M
low_impulse41(ki,:)=dyma(1,.,4,2,ki);
end
low_impulsepct41=prctile(low_impulse41,[2.5 50 97.5],1)';
low_impulse42=zeros(M,nh+1);
for ki=1:M
low_impulse42(ki,:)=dyma(2,.,4,2,ki);
end
low_impulsepct42=prctile(low_impulse42,[2.5 50 97.5],1)';
low_impulse43=zeros(M,nh+1);
for ki=1:M
low_impulse43(ki,:)=dyma(3,.,4,2,ki);
end
low_impulsepct43=prctile(low_impulse43,[2.5 50 97.5],1)';
toc;
figure(1);
subplot(3,2,1);
plot(high_impulsepct11);
hold;
plot(low_impulsepct11);
subplot(3,2,2);
plot(high_impulsepct21);
hold;
plot(low_impulsepct21);
subplot(3,2,3);
plot(high_impulsepct12);
hold;

```

```
plot(low_impulsepct12);
subplot(3,2,4);
plot(high_impulsepct22);
hold;
plot(low_impulsepct22);
subplot(3,2,5);
plot(high_impulsepct13);
hold;
plot(low_impulsepct13);
subplot(3,2,6);
plot(high_impulsepct23);
hold;
plot(low_impulsepct23);
figure(2);
subplot(3,2,1);
plot(high_impulsepct31);
hold;
plot(low_impulsepct31);
subplot(3,2,2);
plot(high_impulsepct41);
hold;
plot(low_impulsepct41);
subplot(3,2,3);
plot(high_impulsepct32);
hold;
plot(low_impulsepct32);
subplot(3,2,4);
```



```

plot(high_impulsepct42);
hold;
plot(low_impulsepct42);
subplot(3,2,5);
plot(high_impulsepct33);
hold;
plot(low_impulsepct33);
subplot(3,2,6);
plot(high_impulsepct43);
hold;
plot(low_impulsepct43);

```

A.3 Parts of Stata Code Used in Chapter 3

```

// 9 Models: Riskmet, GARCH, GARCHt, TARCH, TARCHt, APARCH, APARCHt, EGARCHt
log using C:\Users\Febrio\Documents\Dissertation\3rd_Paper\DataAnalysis\log2_8models_///
with_outofsamplpredict.log, replace
use "C:\Users\Febrio\Documents\Dissertation\3rd_Paper\DataAnalysis\fivemarket3071.dta" ///
if day(1/2821), clear
tsset day
set more off
eststo clear
scalar drop _all
local markets 'x' "axko hsi ftse bvsp gspc"
foreach x of local markets {
//1. Riskmetrics (EMWA)
//constraints: (1) alpha+beta=1 and (2) gamma=0

```

```

constraint 1 [ARCH]_b[L.arch]+[ARCH]_b[L.garch]=1
constraint 2 ['x']_b[_cons]=0
arch 'x', arch(1) garch(1) constraints(1,2)
eststo 'x'_riskmet
predict 'x'_riskmet_m,xb
predict 'x'_riskmet_v,variance
gen 'x'_riskmet_h='x'_riskmet_v^(1/2)
gen 'x'_riskmet_var95='x'_riskmet_m-(1.645*'x'_riskmet_h)
gen 'x'_riskmet_var99='x'_riskmet_m-(2.326*'x'_riskmet_h)
//Backtests
//Kuipec
//VaR95
egen 'x'_riskmet_var95_noexceed2= count('x') if 'x' > 'x'_riskmet_var95 & 'x' < . & tin(2572,3071)
ameans 'x'_riskmet_var95_noexceed2
scalar 'x'_riskmet_var95_noexceed_out=r(N)
egen 'x'_riskmet_var95_exceed2= count('x') if 'x' < 'x'_riskmet_var95 & 'x' < . & tin(2572,3071)
ameans 'x'_riskmet_var95_exceed2
scalar 'x'_riskmet_var95_exceed_out=r(N)
scalar 'x'_riskmet_var95_N_out='x'_riskmet_var95_noexceed_out + 'x'_riskmet_var95_exceed_out
scalar 'x'_riskmet_var95_LMk_out=-2*ln((.95^'x'_riskmet_var95_noexceed_out*.05///
^('x'_riskmet_var95_N_out-'x'_riskmet_var95_noexceed_out))///
/((1-((('x'_riskmet_var95_N_out-'x'_riskmet_var95_noexceed_out)/'x'_riskmet_var95_N_out))///
^'x'_riskmet_var95_noexceed_out///
*((('x'_riskmet_var95_N_out-'x'_riskmet_var95_noexceed_out)/'x'_riskmet_var95_N_out)///
^('x'_riskmet_var95_N_out-'x'_riskmet_var95_noexceed_out)))
scalar 'x'_riskmet_var95_LMk_P_out=chi2tail(1,'x'_riskmet_var95_LMk_out)
//VaR99

```

```

egen 'x'_riskmet_var99_noexceed2= count('x') if 'x' > 'x'_riskmet_var99 & 'x' < . & tin(2572,3071)
ameans 'x'_riskmet_var99_noexceed2
scalar 'x'_riskmet_var99_noexceed_out=r(N)
egen 'x'_riskmet_var99_exceed2= count('x') if 'x' < 'x'_riskmet_var99 & 'x' < . & tin(2572,3071)
ameans 'x'_riskmet_var99_exceed2
scalar 'x'_riskmet_var99_exceed_out=r(N)
scalar 'x'_riskmet_var99_N_out='x'_riskmet_var99_noexceed_out + 'x'_riskmet_var99_exceed_out
scalar 'x'_riskmet_var99_LMk_out=-2*ln((.99^'x'_riskmet_var99_noexceed_out*.01///
^('x'_riskmet_var99_N_out-'x'_riskmet_var99_noexceed_out))///
/((1-((('x'_riskmet_var99_N_out-'x'_riskmet_var99_noexceed_out)/'x'_riskmet_var99_N_out))///
^'x'_riskmet_var99_noexceed_out///
*((('x'_riskmet_var99_N_out-'x'_riskmet_var99_noexceed_out)/'x'_riskmet_var99_N_out)///
^('x'_riskmet_var99_N_out-'x'_riskmet_var99_noexceed_out)))
scalar 'x'_riskmet_var99_LMk_P_out=chi2tail(1,'x'_riskmet_var99_LMk_out)
//Loss function
gen 'x'_riskmet_er=('x'_riskmet_h-'x')
gen 'x'_riskmet_er2='x'_riskmet_er^2
sum 'x'_riskmet_er2 if tin(2572,3071)
scalar 'x'_riskmet_SSE_out = r(sum)
scalar 'x'_riskmet_MSE_out = 'x'_riskmet_SSE_out/r(N)
scalar 'x'_riskmet_RMSE_out= sqrt('x'_riskmet_MSE_out)
//2. GARCH(1,1) Bollerslev1986
arch 'x', arch(1) garch(1)
eststo 'x'_garch11
predict 'x'_garch11_m,xb
predict 'x'_garch11_v,variance
gen 'x'_garch11_h='x'_garch11_v^(1/2)

```

```

gen 'x'_garch11_var95='x'_garch11_m-(1.645*'x'_garch11_h)
gen 'x'_garch11_var99='x'_garch11_m-(2.326*'x'_garch11_h)

//Backtests

//Kuipec

//VaR95

egen 'x'_garch11_var95_noexceed2= count('x') if 'x' > 'x'_garch11_var95 & 'x' < . & tin(2572,3071)
ameans 'x'_garch11_var95_noexceed2

scalar 'x'_garch11_var95_noexceed_out=r(N)

egen 'x'_garch11_var95_exceed2= count('x') if 'x' < 'x'_garch11_var95 & 'x' < . & tin(2572,3071)
ameans 'x'_garch11_var95_exceed2

scalar 'x'_garch11_var95_exceed_out=r(N)

scalar 'x'_garch11_var95_N_out='x'_garch11_var95_noexceed_out + 'x'_garch11_var95_exceed_out
scalar 'x'_garch11_var95_LMk_out=-2*ln((.95^'x'_garch11_var95_noexceed_out*.05///
^('x'_garch11_var95_N_out-'x'_garch11_var95_noexceed_out))///
/((1-((('x'_garch11_var95_N_out-'x'_garch11_var95_noexceed_out)/'x'_garch11_var95_N_out))///
^'x'_garch11_var95_noexceed_out*((('x'_garch11_var95_N_out-'x'_garch11_var95_noexceed_out))///
/'x'_garch11_var95_N_out)^('x'_garch11_var95_N_out-'x'_garch11_var95_noexceed_out)))

scalar 'x'_garch11_var95_LMk_P_out=chi2tail(1,'x'_garch11_var95_LMk_out)

//VaR99

egen 'x'_garch11_var99_noexceed2= count('x') if 'x' > 'x'_garch11_var99 & 'x' < . & tin(2572,3071)
ameans 'x'_garch11_var99_noexceed2

scalar 'x'_garch11_var99_noexceed_out=r(N)

egen 'x'_garch11_var99_exceed2= count('x') if 'x' < 'x'_garch11_var99 & 'x' < . & tin(2572,3071)
ameans 'x'_garch11_var99_exceed2

scalar 'x'_garch11_var99_exceed_out=r(N)

scalar 'x'_garch11_var99_N_out='x'_garch11_var99_noexceed_out + 'x'_garch11_var99_exceed_out
scalar 'x'_garch11_var99_LMk_out=-2*ln((.99^'x'_garch11_var99_noexceed_out*.01///

```

```

^('x'_garch11_var99_N_out-'x'_garch11_var99_noexceed_out))//
/((1-((('x'_garch11_var99_N_out-'x'_garch11_var99_noexceed_out)/'x'_garch11_var99_N_out))//
^'x'_garch11_var99_noexceed_out*((('x'_garch11_var99_N_out-'x'_garch11_var99_noexceed_out))//
/'x'_garch11_var99_N_out)^('x'_garch11_var99_N_out-'x'_garch11_var99_noexceed_out)))
scalar 'x'_garch11_var99_LMk_P_out=chi2tail(1,'x'_garch11_var99_LMk_out)
//Loss function
gen 'x'_garch11_er=('x'_garch11_h-'x')
gen 'x'_garch11_er2='x'_garch11_er^2
sum 'x'_garch11_er2 if tin(2572,3071)
scalar 'x'_garch11_SSE_out = r(sum)
scalar 'x'_garch11_MSE_out = 'x'_garch11_SSE_out/r(N)
scalar 'x'_garch11_RMSE_out= sqrt('x'_garch11_MSE_out)
//3. GARCH(1,1) t Bollerslev1986
arch 'x', arch(1) garch(1) dist(t)
eststo 'x'_garch11t
predict 'x'_garch11t_m,xb
predict 'x'_garch11t_v,variance
gen 'x'_garch11t_h='x'_garch11t_v^(1/2)
gen 'x'_garch11t_var95='x'_garch11t_m-(invttail(e(tdf),.05)*'x'_garch11t_h)
gen 'x'_garch11t_var99='x'_garch11t_m-(invttail(e(tdf),.01)*'x'_garch11t_h)
//Backtests
//Kuipec
//VaR95
egen 'x'_garch11t_var95_noexceed2= count('x') if 'x' > 'x'_garch11t_var95 & 'x' < . &
tin(2572,3071)
ameans 'x'_garch11t_var95_noexceed2
scalar 'x'_garch11t_var95_noexceed_out=r(N)

```

```

egen 'x'_garch11t_var95_exceed2= count('x') if 'x' < 'x'_garch11t_var95 & 'x' < . & tin(2572,3071)
ameans 'x'_garch11t_var95_exceed2
scalar 'x'_garch11t_var95_exceed_out=r(N)
scalar 'x'_garch11t_var95_N_out='x'_garch11t_var95_noexceed_out + 'x'_garch11t_var95_exceed_out
scalar 'x'_garch11t_var95_LMk_out=-2*ln((.95^'x'_garch11t_var95_noexceed_out*.05///
^('x'_garch11t_var95_N_out-'x'_garch11t_var95_noexceed_out))///
/((1-((('x'_garch11t_var95_N_out-'x'_garch11t_var95_noexceed_out)/'x'_garch11t_var95_N_out))///
^'x'_garch11t_var95_noexceed_out*(('x'_garch11t_var95_N_out-'x'_garch11t_var95_noexceed_out))///
/'x'_garch11t_var95_N_out)^(('x'_garch11t_var95_N_out-'x'_garch11t_var95_noexceed_out)))
scalar 'x'_garch11t_var95_LMk_P_out=chi2tail(1,'x'_garch11t_var95_LMk_out)
//VaR99
egen 'x'_garch11t_var99_noexceed2= count('x') if 'x' > 'x'_garch11t_var99 & 'x' < . &
tin(2572,3071)
ameans 'x'_garch11t_var99_noexceed2
scalar 'x'_garch11t_var99_noexceed_out=r(N)
egen 'x'_garch11t_var99_exceed2= count('x') if 'x' < 'x'_garch11t_var99 & 'x' < . & tin(2572,3071)
ameans 'x'_garch11t_var99_exceed2
scalar 'x'_garch11t_var99_exceed_out=r(N)
scalar 'x'_garch11t_var99_N_out='x'_garch11t_var99_noexceed_out + 'x'_garch11t_var99_exceed_out
scalar 'x'_garch11t_var99_LMk_out=-2*ln((.99^'x'_garch11t_var99_noexceed_out*.01///
^('x'_garch11t_var99_N_out-'x'_garch11t_var99_noexceed_out))///
/((1-((('x'_garch11t_var99_N_out-'x'_garch11t_var99_noexceed_out)/'x'_garch11t_var99_N_out))///
^'x'_garch11t_var99_noexceed_out*(('x'_garch11t_var99_N_out-'x'_garch11t_var99_noexceed_out))///
/'x'_garch11t_var99_N_out)^(('x'_garch11t_var99_N_out-'x'_garch11t_var99_noexceed_out)))
scalar 'x'_garch11t_var99_LMk_P_out=chi2tail(1,'x'_garch11t_var99_LMk_out)
//Loss function
gen 'x'_garch11t_er=('x'_garch11t_h-'x')

```

```

gen 'x'_garch11t_er2='x'_garch11t_er^2
sum 'x'_garch11t_er2 if tin(2572,3071)
scalar 'x'_garch11t_SSE_out = r(sum)
scalar 'x'_garch11t_MSE_out = 'x'_garch11t_SSE_out/r(N)
scalar 'x'_garch11t_RMSE_out= sqrt('x'_garch11t_MSE_out)
//4. TARCH(1,1) GJR1993
arch 'x', arch(1) garch(1) tarch(1)
eststo 'x'_tarch11
predict 'x'_tarch11_m,xb
predict 'x'_tarch11_v,variance
gen 'x'_tarch11_h='x'_tarch11_v^(1/2)
gen 'x'_tarch11_var95='x'_tarch11_m-(1.645*'x'_tarch11_h)
gen 'x'_tarch11_var99='x'_tarch11_m-(2.326*'x'_tarch11_h)
//Backtests
//Kuipec
//VaR95
egen 'x'_tarch11_var95_noexceed2= count('x') if 'x' > 'x'_tarch11_var95 & 'x' < . & tin(2572,3071)
ameans 'x'_tarch11_var95_noexceed2
scalar 'x'_tarch11_var95_noexceed_out=r(N)
egen 'x'_tarch11_var95_exceed2= count('x') if 'x' < 'x'_tarch11_var95 & 'x' < . & tin(2572,3071)
ameans 'x'_tarch11_var95_exceed2
scalar 'x'_tarch11_var95_exceed_out=r(N)
scalar 'x'_tarch11_var95_N_out='x'_tarch11_var95_noexceed_out + 'x'_tarch11_var95_exceed_out
scalar 'x'_tarch11_var95_LMk_out=-2*ln((.95^'x'_tarch11_var95_noexceed_out*.05///
^('x'_tarch11_var95_N_out-'x'_tarch11_var95_noexceed_out))///
/((1-((('x'_tarch11_var95_N_out-'x'_tarch11_var95_noexceed_out)/'x'_tarch11_var95_N_out))///
^'x'_tarch11_var95_noexceed_out*(('x'_tarch11_var95_N_out-'x'_tarch11_var95_noexceed_out))///

```

```

/'x'_tarch11_var95_N_out)^(('x'_tarch11_var95_N_out-'x'_tarch11_var95_noexceed_out)))
scalar 'x'_tarch11_var95_LMk_P_out=chi2tail(1,'x'_tarch11_var95_LMk_out)
//VaR99
egen 'x'_tarch11_var99_noexceed2= count('x') if 'x' > 'x'_tarch11_var99 & 'x' < . & tin(2572,3071)
ameans 'x'_tarch11_var99_noexceed2
scalar 'x'_tarch11_var99_noexceed_out=r(N)
egen 'x'_tarch11_var99_exceed2= count('x') if 'x' < 'x'_tarch11_var99 & 'x' < . & tin(2572,3071)
ameans 'x'_tarch11_var99_exceed2
scalar 'x'_tarch11_var99_exceed_out=r(N)
scalar 'x'_tarch11_var99_N_out='x'_tarch11_var99_noexceed_out + 'x'_tarch11_var99_exceed_out
scalar 'x'_tarch11_var99_LMk_out=-2*ln((.99^('x'_tarch11_var99_noexceed_out*.01///
^('x'_tarch11_var99_N_out-'x'_tarch11_var99_noexceed_out))///
/((1-((('x'_tarch11_var99_N_out-'x'_tarch11_var99_noexceed_out)/('x'_tarch11_var99_N_out))///
^('x'_tarch11_var99_noexceed_out*((('x'_tarch11_var99_N_out-'x'_tarch11_var99_noexceed_out))///
/'x'_tarch11_var99_N_out)^(('x'_tarch11_var99_N_out-'x'_tarch11_var99_noexceed_out)))
scalar 'x'_tarch11_var99_LMk_P_out=chi2tail(1,'x'_tarch11_var99_LMk_out)
//Loss function
gen 'x'_tarch11_er=('x'_tarch11_h-'x')
gen 'x'_tarch11_er2='x'_tarch11_er^2
sum 'x'_tarch11_er2 if tin(2572,3071)
scalar 'x'_tarch11_SSE_out = r(sum)
scalar 'x'_tarch11_MSE_out = 'x'_tarch11_SSE_out/r(N)
scalar 'x'_tarch11_RMSE_out= sqrt('x'_tarch11_MSE_out)
//5. TARCH(1,1) t GJR1993
arch 'x', arch(1) garch(1) tarch(1) dist(t)
eststo 'x'_tarch11t
predict 'x'_tarch11t_m,xb

```



```

predict 'x'_tarch11t_v,variance
gen 'x'_tarch11t_h='x'_tarch11t_v^(1/2)
gen 'x'_tarch11t_var95='x'_tarch11t_m-(invttail(e(tdf),.05)*'x'_tarch11t_h)
gen 'x'_tarch11t_var99='x'_tarch11t_m-(invttail(e(tdf),.01)*'x'_tarch11t_h)
//Backtests
//Kuipec
//VaR95
egen 'x'_tarch11t_var95_noexceed2= count('x') if 'x' > 'x'_tarch11t_var95 & 'x' < . &
tin(2572,3071)
ameans 'x'_tarch11t_var95_noexceed2
scalar 'x'_tarch11t_var95_noexceed_out=r(N)
egen 'x'_tarch11t_var95_exceed2= count('x') if 'x' < 'x'_tarch11t_var95 & 'x' < . & tin(2572,3071)
ameans 'x'_tarch11t_var95_exceed2
scalar 'x'_tarch11t_var95_exceed_out=r(N)
scalar 'x'_tarch11t_var95_N_out='x'_tarch11t_var95_noexceed_out + 'x'_tarch11t_var95_exceed_out
scalar 'x'_tarch11t_var95_LMk_out=-2*ln((.95^'x'_tarch11t_var95_noexceed_out*.05///
^('x'_tarch11t_var95_N_out-'x'_tarch11t_var95_noexceed_out))///
/((1-((('x'_tarch11t_var95_N_out-'x'_tarch11t_var95_noexceed_out)/'x'_tarch11t_var95_N_out))///
^'x'_tarch11t_var95_noexceed_out*((('x'_tarch11t_var95_N_out-'x'_tarch11t_var95_noexceed_out))///
/'x'_tarch11t_var95_N_out)^(('x'_tarch11t_var95_N_out-'x'_tarch11t_var95_noexceed_out)))
scalar 'x'_tarch11t_var95_LMk_P_out=chi2tail(1,'x'_tarch11t_var95_LMk_out)
//VaR99
egen 'x'_tarch11t_var99_noexceed2= count('x') if 'x' > 'x'_tarch11t_var99 & 'x' < . &
tin(2572,3071)
ameans 'x'_tarch11t_var99_noexceed2
scalar 'x'_tarch11t_var99_noexceed_out=r(N)
egen 'x'_tarch11t_var99_exceed2= count('x') if 'x' < 'x'_tarch11t_var99 & 'x' < . & tin(2572,3071)

```

```

ameans 'x'_tarch11t_var99_exceed2
scalar 'x'_tarch11t_var99_exceed_out=r(N)
scalar 'x'_tarch11t_var99_N_out='x'_tarch11t_var99_noexceed_out + 'x'_tarch11t_var99_exceed_out
scalar 'x'_tarch11t_var99_LMk_out=-2*ln((.99^'x'_tarch11t_var99_noexceed_out*.01///
^('x'_tarch11t_var99_N_out-'x'_tarch11t_var99_noexceed_out))///
/((1-((('x'_tarch11t_var99_N_out-'x'_tarch11t_var99_noexceed_out)/'x'_tarch11t_var99_N_out))///
^'x'_tarch11t_var99_noexceed_out*((('x'_tarch11t_var99_N_out-'x'_tarch11t_var99_noexceed_out))///
/'x'_tarch11t_var99_N_out)^('x'_tarch11t_var99_N_out-'x'_tarch11t_var99_noexceed_out)))
scalar 'x'_tarch11t_var99_LMk_P_out=chi2tail(1,'x'_tarch11t_var99_LMk_out)
//Loss function
gen 'x'_tarch11t_er=('x'_tarch11t_h-'x')
gen 'x'_tarch11t_er2='x'_tarch11t_er^2
sum 'x'_tarch11t_er2 if tin(2572,3071)
scalar 'x'_tarch11t_SSE_out = r(sum)
scalar 'x'_tarch11t_MSE_out = 'x'_tarch11t_SSE_out/r(N)
scalar 'x'_tarch11t_RMSE_out= sqrt('x'_tarch11t_MSE_out)
//6. PARCH(1,1) Higgins and Bera 1992
arch 'x', parch(1) pgarch(1)
eststo 'x'_parch11
predict 'x'_parch11_m,xb
predict 'x'_parch11_v,variance
gen 'x'_parch11_h='x'_parch11_v^(1/2)
gen 'x'_parch11_var95='x'_parch11_m-(1.645*'x'_parch11_h)
gen 'x'_parch11_var99='x'_parch11_m-(2.326*'x'_parch11_h)
//Backtests
//Kuipec
//VaR95

```

```

egen 'x'_parch11_var95_noexceed2= count('x') if 'x' > 'x'_parch11_var95 & 'x' < . &
tin(2572,3071)

ameans 'x'_parch11_var95_noexceed2

scalar 'x'_parch11_var95_noexceed_out=r(N)

egen 'x'_parch11_var95_exceed2= count('x') if 'x' < 'x'_parch11_var95 & 'x' < . & tin(2572,3071)

ameans 'x'_parch11_var95_exceed2

scalar 'x'_parch11_var95_exceed_out=r(N)

scalar 'x'_parch11_var95_N_out='x'_parch11_var95_noexceed_out + 'x'_parch11_var95_exceed_out

scalar 'x'_parch11_var95_LMk_out=-2*ln((.95^'x'_parch11_var95_noexceed_out*.05///
^('x'_parch11_var95_N_out-'x'_parch11_var95_noexceed_out))///
/((1-('x'_parch11_var95_N_out-'x'_parch11_var95_noexceed_out)/'x'_parch11_var95_N_out))///
^'x'_parch11_var95_noexceed_out*(('x'_parch11_var95_N_out-'x'_parch11_var95_noexceed_out)///
/'x'_parch11_var95_N_out)^('x'_parch11_var95_N_out-'x'_parch11_var95_noexceed_out)))

scalar 'x'_parch11_var95_LMk_P_out=chi2tail(1,'x'_parch11_var95_LMk_out)

//VaR99

egen 'x'_parch11_var99_noexceed2= count('x') if 'x' > 'x'_parch11_var99 & 'x' < . &
tin(2572,3071)

ameans 'x'_parch11_var99_noexceed2

scalar 'x'_parch11_var99_noexceed_out=r(N)

egen 'x'_parch11_var99_exceed2= count('x') if 'x' < 'x'_parch11_var99 & 'x' < . & tin(2572,3071)

ameans 'x'_parch11_var99_exceed2

scalar 'x'_parch11_var99_exceed_out=r(N)

scalar 'x'_parch11_var99_N_out='x'_parch11_var99_noexceed_out + 'x'_parch11_var99_exceed_out

scalar 'x'_parch11_var99_LMk_out=-2*ln((.99^'x'_parch11_var99_noexceed_out*.01///
^('x'_parch11_var99_N_out-'x'_parch11_var99_noexceed_out))///
/((1-('x'_parch11_var99_N_out-'x'_parch11_var99_noexceed_out)/'x'_parch11_var99_N_out))///
^'x'_parch11_var99_noexceed_out*(('x'_parch11_var99_N_out-'x'_parch11_var99_noexceed_out)///

```

```

/('x'_parch11_var99_N_out)^('x'_parch11_var99_N_out-'x'_parch11_var99_noexceed_out)))
scalar 'x'_parch11_var99_LMk_P_out=chi2tail(1,'x'_parch11_var99_LMk_out)
//Loss function
gen 'x'_parch11_er=('x'_parch11_h-'x')
gen 'x'_parch11_er2='x'_parch11_er^2
sum 'x'_parch11_er2 if tin(2572,3071)
scalar 'x'_parch11_SSE_out = r(sum)
scalar 'x'_parch11_MSE_out = 'x'_parch11_SSE_out/r(N)
scalar 'x'_parch11_RMSE_out= sqrt('x'_parch11_MSE_out)
//7. parch (1,1) t Higgins and Bera 1992
arch 'x', parch(1) pgarch(1) distribution(t)
eststo 'x'_parch11t
predict 'x'_parch11t_m,xb
predict 'x'_parch11t_v,variance
gen 'x'_parch11t_h='x'_parch11t_v^(1/2)
gen 'x'_parch11t_var95='x'_parch11t_m-(invttail(e(tdf),.05)*'x'_parch11t_h)
gen 'x'_parch11t_var99='x'_parch11t_m-(invttail(e(tdf),.01)*'x'_parch11t_h)
//Backtests
//Kuipec
//VaR95
egen 'x'_parch11t_var95_noexceed2= count('x') if 'x' > 'x'_parch11t_var95 & 'x' < . &
tin(2572,3071)
ameans 'x'_parch11t_var95_noexceed2
scalar 'x'_parch11t_var95_noexceed_out=r(N)
egen 'x'_parch11t_var95_exceed2= count('x') if 'x' < 'x'_parch11t_var95 & 'x' < . & tin(2572,3071)
ameans 'x'_parch11t_var95_exceed2
scalar 'x'_parch11t_var95_exceed_out=r(N)

```

```

scalar 'x'_parch11t_var95_N_out='x'_parch11t_var95_noexceed_out + 'x'_parch11t_var95_exceed_out
scalar 'x'_parch11t_var95_LMk_out=-2*ln((.95^'x'_parch11t_var95_noexceed_out*.05///
^('x'_parch11t_var95_N_out-'x'_parch11t_var95_noexceed_out))///
/((1-((('x'_parch11t_var95_N_out-'x'_parch11t_var95_noexceed_out)/'x'_parch11t_var95_N_out))///
^'x'_parch11t_var95_noexceed_out*((('x'_parch11t_var95_N_out-'x'_parch11t_var95_noexceed_out))///
/'x'_parch11t_var95_N_out)^(('x'_parch11t_var95_N_out-'x'_parch11t_var95_noexceed_out)))
scalar 'x'_parch11t_var95_LMk_P_out=chi2tail(1,'x'_parch11t_var95_LMk_out)
//VaR99
egen 'x'_parch11t_var99_noexceed2= count('x') if 'x' > 'x'_parch11t_var99 & 'x' < . &
tin(2572,3071)
ameans 'x'_parch11t_var99_noexceed2
scalar 'x'_parch11t_var99_noexceed_out=r(N)
egen 'x'_parch11t_var99_exceed2= count('x') if 'x' < 'x'_parch11t_var99 & 'x' < . & tin(2572,3071)
ameans 'x'_parch11t_var99_exceed2
scalar 'x'_parch11t_var99_exceed_out=r(N)
scalar 'x'_parch11t_var99_N_out='x'_parch11t_var99_noexceed_out + 'x'_parch11t_var99_exceed_out
scalar 'x'_parch11t_var99_LMk_out=-2*ln((.99^'x'_parch11t_var99_noexceed_out*.01///
^('x'_parch11t_var99_N_out-'x'_parch11t_var99_noexceed_out))///
/((1-((('x'_parch11t_var99_N_out-'x'_parch11t_var99_noexceed_out)/'x'_parch11t_var99_N_out))///
^'x'_parch11t_var99_noexceed_out*((('x'_parch11t_var99_N_out-'x'_parch11t_var99_noexceed_out))///
/'x'_parch11t_var99_N_out)^(('x'_parch11t_var99_N_out-'x'_parch11t_var99_noexceed_out)))
scalar 'x'_parch11t_var99_LMk_P_out=chi2tail(1,'x'_parch11t_var99_LMk_out)
//Loss function
gen 'x'_parch11t_er=('x'_parch11t_h-'x')
gen 'x'_parch11t_er2='x'_parch11t_er^2
sum 'x'_parch11t_er2 if tin(2572,3071)
scalar 'x'_parch11t_SSE_out = r(sum)

```

```

scalar 'x'_parch11t_MSE_out = 'x'_parch11t_SSE_out/r(N)
scalar 'x'_parch11t_RMSE_out= sqrt('x'_parch11t_MSE_out)
//8. EGARCH (1,1) t Nelson 1991
arch 'x', earch(1) egarch(1) dist(t)
eststo 'x'_egarcht
predict 'x'_egarcht_m,xb
predict 'x'_egarcht_v,variance
gen 'x'_egarcht_h='x'_egarcht_v^(1/2)
gen 'x'_egarcht_var95='x'_egarcht_m-(invttail(e(tdf),.05)*'x'_egarcht_h)
gen 'x'_egarcht_var99='x'_egarcht_m-(invttail(e(tdf),.01)*'x'_egarcht_h)
//Backtests
//Kuipec
//VaR95
egen 'x'_egarcht_var95_noexceed2= count('x') if 'x' > 'x'_egarcht_var95 & 'x' < . & tin(2572,3071)
ameans 'x'_egarcht_var95_noexceed2
scalar 'x'_egarcht_var95_noexceed_out=r(N)
egen 'x'_egarcht_var95_exceed2= count('x') if 'x' < 'x'_egarcht_var95 & 'x' < . & tin(2572,3071)
ameans 'x'_egarcht_var95_exceed2
scalar 'x'_egarcht_var95_exceed_out=r(N)
scalar 'x'_egarcht_var95_N_out='x'_egarcht_var95_noexceed_out + 'x'_egarcht_var95_exceed_out
scalar 'x'_egarcht_var95_LMk_out=-2*ln((.95^'x'_egarcht_var95_noexceed_out*.05///
^('x'_egarcht_var95_N_out-'x'_egarcht_var95_noexceed_out))///
/((1-((('x'_egarcht_var95_N_out-'x'_egarcht_var95_noexceed_out)/'x'_egarcht_var95_N_out))///
^'x'_egarcht_var95_noexceed_out*((('x'_egarcht_var95_N_out-'x'_egarcht_var95_noexceed_out))///
/'x'_egarcht_var95_N_out)^('x'_egarcht_var95_N_out-'x'_egarcht_var95_noexceed_out)))
scalar 'x'_egarcht_var95_LMk_P_out=chi2tail(1,'x'_egarcht_var95_LMk_out)
//VaR99

```

```

egen 'x'_egarcht_var99_noexceed2= count('x') if 'x' > 'x'_egarcht_var99 & 'x' < . & tin(2572,3071)
ameans 'x'_egarcht_var99_noexceed2
scalar 'x'_egarcht_var99_noexceed_out=r(N)
egen 'x'_egarcht_var99_exceed2= count('x') if 'x' < 'x'_egarcht_var99 & 'x' < . & tin(2572,3071)
ameans 'x'_egarcht_var99_exceed2
scalar 'x'_egarcht_var99_exceed_out=r(N)
scalar 'x'_egarcht_var99_N_out='x'_egarcht_var99_noexceed_out + 'x'_egarcht_var99_exceed_out
scalar 'x'_egarcht_var99_LMk_out=-2*ln((.99^'x'_egarcht_var99_noexceed_out*.01///
^('x'_egarcht_var99_N_out-'x'_egarcht_var99_noexceed_out))///
/((1-((('x'_egarcht_var99_N_out-'x'_egarcht_var99_noexceed_out)/'x'_egarcht_var99_N_out))///
^'x'_egarcht_var99_noexceed_out*((('x'_egarcht_var99_N_out-'x'_egarcht_var99_noexceed_out)///
/'x'_egarcht_var99_N_out)^('x'_egarcht_var99_N_out-'x'_egarcht_var99_noexceed_out)))
scalar 'x'_egarcht_var99_LMk_P_out=chi2tail(1,'x'_egarcht_var99_LMk_out)
//Loss function
gen 'x'_egarcht_er=('x'_egarcht_h-'x')
gen 'x'_egarcht_er2='x'_egarcht_er^2
sum 'x'_egarcht_er2 if tin(2572,3071)
scalar 'x'_egarcht_SSE_out = r(sum)
scalar 'x'_egarcht_MSE_out = 'x'_egarcht_SSE_out/r(N)
scalar 'x'_egarcht_RMSE_out= sqrt('x'_egarcht_MSE_out)
esttab 'x'_riskmet 'x'_garch11 'x'_garch11t 'x'_tarch11 'x'_tarch11t 'x'_parch11 'x'_parch11t
///
'x'_egarcht, scalars(ll) csv staraux
}
scalar list
log close

```